

SHARP

Worksheet 12 Memorandum: Revision Term 1

Grade 11 Mathematics

1. a)
$$\frac{x^2}{y^2} \div \frac{(xy)^0}{(xy^3)^2}$$
$$= \frac{x^2}{y^2} \times \frac{x^2 y^6}{1}$$
$$= \frac{x^4 y^6}{y^2}$$
$$= x^4 y^4$$

b)
$$\frac{\sqrt{4x^6 y^2}}{x^3 y^{-4}} \times \frac{x^2 y^3}{(x^2 y^{-3})^{-1}}$$
$$= \frac{2x^3 y}{x^3 y^{-4}} \times \frac{x^2 y^3}{x^{-2} y^3}$$
$$= \frac{2y^5}{1} \times \frac{x^4}{1}$$
$$= 2x^4 y^5$$

c)
$$\left(\frac{x}{y} - \frac{1}{x}\right)^{-1}$$
$$= \left(\frac{x^2 - y}{xy}\right)^{-1}$$
$$= \frac{xy}{x^2 - y}$$

d)
$$\sqrt{4\sqrt{8\sqrt{4x^8}}}$$
$$= \sqrt{4\sqrt{8} \times 2x^4}$$
$$= \sqrt{4\sqrt{16x^4}}$$
$$= \sqrt{4 \times 4x^2}$$
$$= \sqrt{16x^2}$$
$$= 4x$$

e)
$$\left(\frac{x}{y^3}\right)^3 \left(\frac{x^2}{y}\right)^{-2} \left(\frac{xy^3}{x^0 y}\right)^4$$
$$= \frac{x^3}{y^9} \times \frac{y^2}{x^4} \times \frac{x^4 y^{12}}{y^4}$$
$$= \frac{x^7 y^{14}}{x^4 y^{13}}$$
$$= x^3 y$$

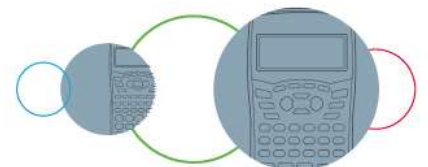
b)
$$\frac{\sqrt{72} - \sqrt{50}}{\sqrt{6}}$$
$$= \frac{6\sqrt{2} - 5\sqrt{2}}{\sqrt{3} \times \sqrt{2}}$$
$$= \frac{\sqrt{2}}{\sqrt{3} \times \sqrt{2}}$$
$$= \frac{1}{\sqrt{3}}$$

2. a)
$$\frac{\sqrt{48} + \sqrt{75}}{\sqrt{27}}$$
$$= \frac{4\sqrt{3} + 5\sqrt{3}}{3\sqrt{3}}$$
$$= \frac{9\sqrt{3}}{3\sqrt{3}}$$
$$= 3$$

d)
$$\sqrt[3]{\sqrt{3600x^{12}} + \sqrt{16x^{12}}}$$
$$= \sqrt[3]{60x^6 + 4x^6}$$
$$= \sqrt[3]{64x^6}$$
$$= 4x^2$$

c)
$$\frac{\sqrt{63} + \sqrt{28}}{\sqrt{150}} \times \frac{\sqrt{384} - \sqrt{294}}{\sqrt{112}}$$
$$= \frac{3\sqrt{7} + 2\sqrt{7}}{5\sqrt{6}} \times \frac{8\sqrt{6} - 7\sqrt{6}}{4\sqrt{7}}$$
$$= \frac{5\sqrt{7}}{5\sqrt{6}} \times \frac{\sqrt{6}}{4\sqrt{7}} = \frac{1}{4}$$

e)
$$\frac{(x^3 y^2)^{\frac{1}{2}}}{(x^2 y^3)^{\frac{1}{3}}} = \frac{x^{\frac{3}{2}} y}{x^{\frac{2}{3}} y^{\frac{1}{3}}} = x^{\frac{3}{2} - \frac{2}{3}} = x^{\frac{5}{6}}$$



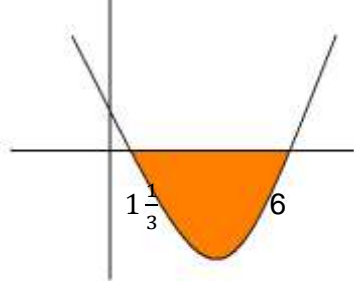
3. a) $3x(x - 2) \leq 4(4x - 6)$

$$3x^2 - 6x \leq 16x - 24$$

$$3x^2 - 6x - 16x + 24 \leq 0$$

$$3x^2 - 22x + 24 \leq 0$$

$$(3x - 4)(x - 6) \leq 0$$



$$\therefore 1\frac{1}{3} \leq x \leq 6$$

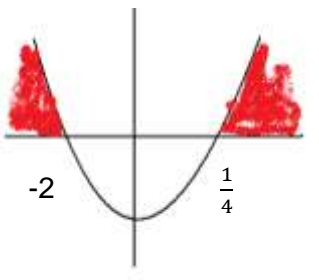
d) $4x(x + 1) \geq 2 - 3x$

$$4x^2 + 4x \geq 2 - 3x$$

$$4x^2 + 4x + 3x - 2 \geq 0$$

$$4x^2 + 7x - 2 \geq 0$$

$$(4x - 1)(x + 2) \geq 0$$



$$x \leq -2 \quad \text{or} \quad x \geq \frac{1}{4}$$

f) $3^{x+3} - 3^{x-1} = 240$

$$3^x(3^3 - 3^{-1}) = 240$$

$$3^x \left(27 - \frac{1}{3}\right) = 240$$

$$3^x \left(26\frac{2}{3}\right) = 240$$

$$3^x = 9$$

$$3^x = 3^2 \quad \therefore x = 2$$

b) $7x^2 - 5x - 3 = 0$

Cant factorise so use formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(7)(-3)}}{2(7)}$$

$$\therefore x = \frac{5 \pm \sqrt{109}}{14}$$

$$\therefore x = -0.39 \quad \text{or} \quad x = 1.10$$

c) $2x^2 - 5x - 7 = 0$

$$2\left(x^2 - \frac{5}{2}x\right) = 7$$

$$x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{7}{2} + \frac{25}{16}$$

$$\left(x - \frac{5}{4}\right)^2 = 5\frac{1}{16}$$

$$x - \frac{5}{4} = -2\frac{1}{4} \quad \text{or} \quad x - \frac{5}{4} = 2\frac{1}{4}$$

$$\therefore x = -2\frac{1}{4} + \frac{5}{4}$$

$$x = 2\frac{1}{4} + \frac{5}{4}$$

$$\therefore x = -1$$

$$x = 3\frac{1}{2}$$

e) $10x^2 + 28x = 6$

$$10x^2 + 28x - 6 = 0$$

$$2(5x^2 + 14x - 3) = 0$$

$$(5x - 1)(x + 3) = 0$$

$$5x = 1 \quad \text{or} \quad x = -3$$

$$x = \frac{1}{5}$$

g) $5x^2 - 42x + 12 = 0$

Cant factorise:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-42) \pm \sqrt{(-42)^2 - 4(5)(12)}}{2(5)}$$

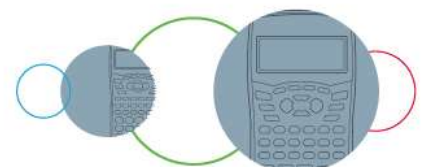
$$x = \frac{21 \pm \sqrt{381}}{5}$$

$$x = 0.30 \quad \text{or} \quad x = 8.1$$

h) $-3x^2 + 4x + 12 = 0$

$$3x^2 - 4x - 12 = 0$$

$$x^2 - \frac{4}{3}x = 4$$



i) $2x(x + 2) = 3(10 - x)$

$$2x^2 + 4x = 30 - 3x$$

$$2x^2 + 4x + 3x - 30 = 0$$

$$2x^2 + 7x - 30 = 0$$

$$(2x - 5)(x + 6) = 0$$

$$2x = 5 \quad \text{or} \quad x = -6$$

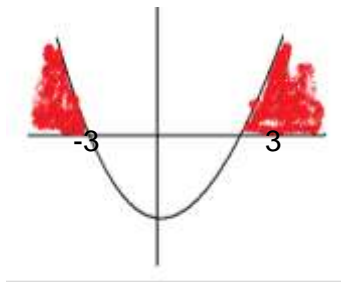
$$x = \frac{5}{2}$$

k) $x^2 - 3x > 3(3 - x)$

$$x^2 - 3x > 9 - 3x$$

$$x^2 - 9 > 0$$

$$(x - 3)(x + 3) > 0$$



$$x < -3 \quad \text{or} \quad x > 3$$

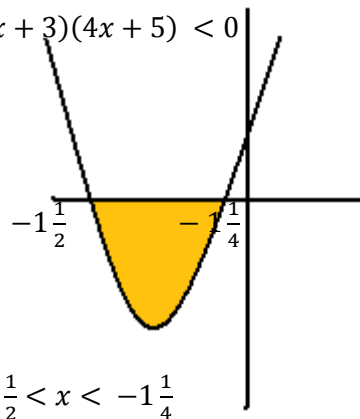
n) $8(x + 1)^2 < -6x - 7$

$$8(x^2 + 2x + 1) + 6x + 7 < 0$$

$$8x^2 + 16x + 8 + 6x + 7 < 0$$

$$8x^2 + 22x + 15 < 0$$

$$(2x + 3)(4x + 5) < 0$$



$$-1\frac{1}{2} < x < -1\frac{1}{4}$$

$$x^2 - \frac{4}{3}x + \frac{4}{9} = 4 + \frac{4}{9}$$

$$\left(x - \frac{2}{3}\right)^2 = 4\frac{4}{9}$$

$$\therefore x - \frac{2}{3} = \frac{-2\sqrt{10}}{3} \quad \text{or} \quad x - \frac{2}{3} = \frac{2\sqrt{10}}{3}$$

$$\therefore x = \frac{2-2\sqrt{10}}{3} \quad x = \frac{2+2\sqrt{10}}{3}$$

$$\therefore x = -1.44 \quad x = 2.77$$

j) $5.2^{x+4} + 5.2^{x-2} = 5\frac{5}{64}$

$$2^x(5.2^4 + 5.2^{-2}) = 5\frac{5}{64}$$

$$2^x\left(80 + \frac{5}{4}\right) = 5\frac{5}{64}$$

$$2^x\left(81\frac{1}{4}\right) = 5\frac{5}{64}$$

$$2^x = \frac{1}{16}$$

$$2^x = 2^{-4}$$

$$x = -4$$

l) $3x^2 + 2 = 5x$

$$3x^2 - 5x + 2 = 0$$

$$(3x - 2)(x - 1) = 0$$

$$3x = 2 \quad \text{or} \quad x = 1$$

$$x = \frac{2}{3}$$

m) $-10x + 2x^2 - 3 = 0$

$$2x^2 - 10x - 3 = 0$$

Cant Factorise:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(-3)}}{2(2)}$$

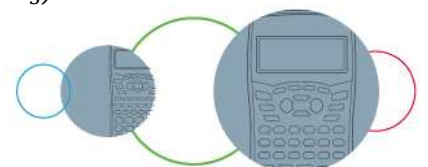
$$x = \frac{5 \pm \sqrt{31}}{2}$$

$$\therefore x = -0.28 \quad \text{or} \quad x = 5.28$$

o) $5^{6x+1} - 5^{6x-1} = 24$

$$5^{6x}(5 - 5^{-1}) = 24$$

$$5^{6x}\left(5 - \frac{1}{5}\right) = 24$$



q) $15x^2 - 13x - 6 = 0$
 $(5x - 6)(3x + 1) = 0$
 $5x = 6$ or $3x = -1$
 $x = \frac{6}{5}$ or $x = -\frac{1}{3}$

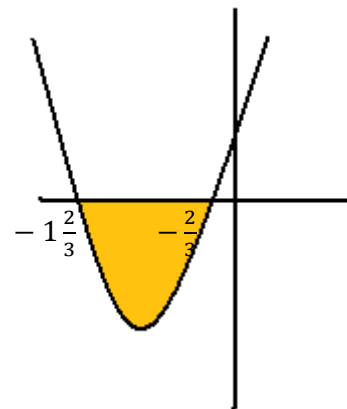
$5^{6x} \left(4\frac{4}{5}\right) = 24$
 $5^{6x} = 5$
 $6x = 1$
 $x = \frac{1}{6}$

r) $-2x^2 + 6x - 4 = 0$
 $x^2 - 3x + 2 = 0$
 $x^2 - 3x + \left(\frac{3}{2}\right)^2 = -2 + \left(\frac{3}{2}\right)^2$
 $\left(x - \frac{3}{2}\right)^2 = \frac{1}{4}$
 $\therefore x - \frac{3}{2} = -\frac{1}{2}$ or $x - \frac{3}{2} = \frac{1}{2}$
 $\therefore x = 1$ or $x = 2$

p) $x^2 - 15x + 17 = 0$
cant factorise
 $\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 $x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(1)(17)}}{2(1)}$
 $\therefore x = \frac{15 \pm \sqrt{157}}{2}$
 $\therefore x = 1.24$ or $x = 13.76$

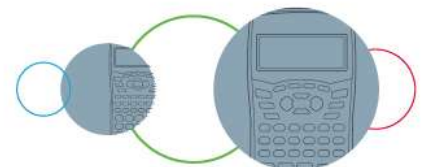
s) $4 \cdot 3^{x+1} + 2 \cdot 3^{x-1} = 4\frac{2}{9}$
 $3^x(4 \cdot 3 + 2 \cdot 3^{-1}) = 4\frac{2}{9}$
 $3^x\left(12 + \frac{2}{3}\right) = 4\frac{2}{9}$
 $3^x\left(12\frac{2}{3}\right) = 4\frac{2}{9}$
 $3^x = \frac{1}{3}$
 $3^x = 3^{-1}$
 $x = -1$

t) $9x(x + 3) \leq 2(3x - 5)$
 $9x^2 + 27x \leq 6x - 10$
 $9x^2 + 27x - 6x + 10 \leq 0$
 $9x^2 + 21x + 10 \leq 0$
 $(3x + 5)(3x + 2) \leq 0$
 $-1\frac{2}{3} \leq x \leq -\frac{2}{3}$



4. a) $\frac{1}{2}y = 4x - 8$ and $y = -x^2 + 9x - 4 \dots 1$
 $y = 8x - 16 \dots 2$
Subs 2 into 1
 $8x - 16 = -x^2 + 9x - 4$
 $x^2 - 9x + 8x - 16 + 4 = 0$
 $x^2 - x - 12 = 0$
 $(x - 4)(x + 3) = 0$
 $x = 4$ or $x = -3$
Subs back into 2:
 $y = 8(4) - 16$ OR $y = 8(-3) - 16$
 $y = 16$ OR $y = -40$

$\therefore (4; 16)$ and $(-3; -40)$



b) $5x + y = 14$ and $y = 2x^2 - 3x + 10 \dots 1$

$y = 14 - 5x \dots 2$

Subs 2 into 1

$14 - 5x = 2x^2 - 3x + 10$

$0 = 2x^2 - 3x + 5x + 10 - 14$

$0 = 2x^2 + 2x - 4$

$0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2$ or $x = 1$

Subs back into 2

$y = 14 - 5(-2)$ OR $y = 14 - 5(1)$

$y = 24$ $y = 9$ $\therefore (-2; 24)$ and $(1; 9)$

c) $4y + 8 = 60x$ and $y = 3x^2 + 6x - 2 \dots 1$

$4y = 60x - 8$

$y = 15x - 2 \dots 2$

Subs 2 into 1

$15x - 2 = 3x^2 + 6x - 2$

$0 = 3x^2 + 6x - 15x - 2 + 2$

$0 = 3x^2 - 9x$

$0 = 3x(x - 3)$

$x = 0$ or $x = 3$

Subs back into 2

$y = 15(0) - 2$ OR $y = 15(3) - 2$

$y = -2$ $y = 43$ $\therefore (0; -2)$ and $(3; 43)$

d) $4 + 3x + y = 0$ and $-\frac{1}{4}y = x^2 + \frac{1}{4}x - 4 \dots 1$

$y = -4 - 3x \dots 2$

Subs 2 into 1

$-\frac{1}{4}(-4 - 3x) = x^2 + \frac{1}{4}x - 4$

$1 + \frac{3}{4}x = x^2 + \frac{1}{4}x - 4$

$0 = x^2 + \frac{1}{4}x - \frac{3}{4}x - 4 - 1$

$0 = x^2 - \frac{2}{4}x - 5$

Subs back into 2

$0 = 2x^2 - x - 10$

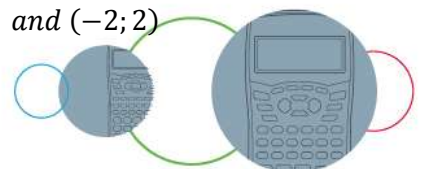
$y = -4 - 3\left(2\frac{1}{2}\right)$ OR $y = -4 - 3(-2)$

$0 = (2x - 5)(x + 2)$

$y = -11\frac{1}{2}$ $y = 2$

$x = \frac{5}{2}$ or $2\frac{1}{2}$ or $x = -2$

$\therefore \left(2\frac{1}{2}; -11\frac{1}{2}\right)$ and $(-2; 2)$



e) $-2y = 24x + 2$ and $y = 2x^2 - 3x + 8 \dots 1$

$y = -12x - 1 \dots 2$

Subs 2 into 1

$-12x - 1 = 2x^2 - 3x + 8$

$0 = 2x^2 - 3x + 12x + 8 + 1$

$0 = 2x^2 + 9x + 9$

$0 = (2x + 3)(x + 3)$

$x = -\frac{3}{2} = -1\frac{1}{2}$ or $x = -3$

Subs back into 2

$y = -12\left(-1\frac{1}{2}\right) - 1$ OR $y = -12(-3) - 1$

$y = 17$

$y = 35$

$\therefore \left(-1\frac{1}{2}; 17\right)$ and $(-3; 35)$

5. a) $\sqrt{-5}$ b) $-2; 2$ and $\frac{4}{3}$ c) $-2; 2$

6. a) $9x^2 + 3 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (0)^2 - 4(9)(3)$

$\Delta = -108$

Roots are non-real

b) $10x^2 + x - 3 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (1)^2 - 4(10)(-3)$

$\Delta = 121 = 11^2$

Roots are real and rational

c) $x^2 + 10x - 11 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (10)^2 - 4(1)(-11)$

$\Delta = 144$

Roots are real and rational

d) $x^2 - 2x + 1 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (-2)^2 - 4(1)(1)$

$\Delta = 0$

Roots are real and equal

e) $5x^2 - 11x + 12 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (-11)^2 - 4(5)(12)$

$\Delta = -119$

Roots are non-real

f) $3x^2 + 5x - 7 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (5)^2 - 4(3)(-7)$

$\Delta = 109$

Roots are real and irrational

g) $2x^2 - 8 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (0)^2 - 4(2)(-8)$

$\Delta = 64$

Roots are real and rational

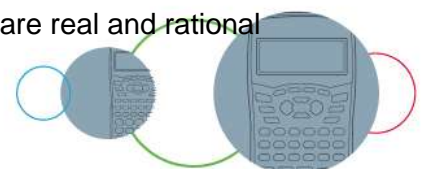
h) $9x^2 + 59x + 30 = 0$

$\Delta = b^2 - 4ac$

$\Delta = (59)^2 - 4(9)(30)$

$\Delta = 2401$

Roots are real and rational



7. a) -6 -12 -18 -24...

Linear

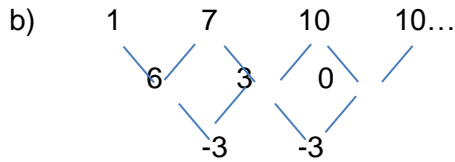
$$\therefore T_n = -6n$$

$$T_n = -6n + b$$

$$\therefore -12 = -6(2) + b$$

$$b = 0$$

$$\text{and } T_{14} = -6(14) = -84$$



$$\therefore T_n = -1\frac{1}{2}n^2 + 10\frac{1}{2}n - 8$$

$$T_{14} = -1\frac{1}{2}(14)^2 + 10\frac{1}{2}(14) - 8$$

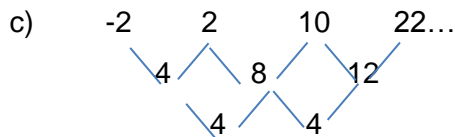
$$T_{14} = -155$$

Quadratic

$$a + b + c = 1 \quad \therefore a = -\frac{3}{2}$$

$$3a + b = 6 \quad \therefore b = 6 - 3\left(-\frac{3}{2}\right) = 10\frac{1}{2}$$

$$2a = -3 \quad c = 1 - \left(-\frac{3}{2}\right) - 10\frac{1}{2} = -8$$



$$a + b + c = -2 \quad \therefore a = 2$$

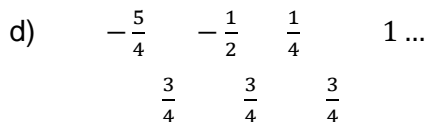
$$3a + b = 4 \quad \therefore b = 4 - 3(2) = -2$$

$$2a = 4 \quad c = -2 - (-2) - 2 = -2 \quad T_n = 2n^2 - 2n - 2$$

$$\therefore T_{14} = 2(14)^2 - 2(14) - 2$$

$$T_{14} = 362$$

Quadratic



$$T_{14} = \frac{3}{4}(14) - 2$$

$$T_{14} = 8\frac{1}{2}$$

Linear

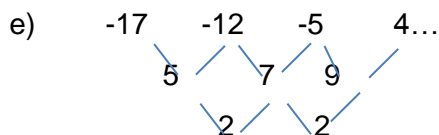
$$T_n = \frac{3}{4}n + b$$

$$-\frac{1}{2} = \frac{3}{4}(2) + b$$

$$-\frac{1}{2} - \frac{3}{2} = b$$

$$b = -2$$

$$T_n = \frac{3}{4}n - 2$$



$$T_{14} = (14)^2 + 2(14) - 20$$

$$T_{14} = 204$$

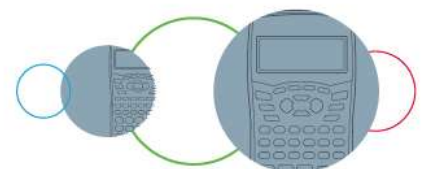
Quadratic

$$a + b + c = -17 \quad a = 1$$

$$3a + b = 5 \quad b = 5 - 3(1) = 2$$

$$2a = 2 \quad c = -17 - 1 - 2 = -20$$

$$T_n = n^2 + 2n - 20$$



8. $y = 2x - 3 \dots 1$ and $25 = y^2 + x^2 \dots 2$

Subs 1 into 2

$$25 = (2x - 3)^2 + x^2$$

$$25 = 4x^2 - 12x + 9 + x^2$$

$$0 = 5x^2 - 12x + 9 - 25$$

$$0 = 5x^2 - 12x - 16$$

Cannot factorise:

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(5)(-16)}}{2(5)}$$

$$\therefore x = \frac{6 \pm 2\sqrt{29}}{5}$$

$$x = -0.95 \text{ or } x = 3.35$$

Subs back into 1:

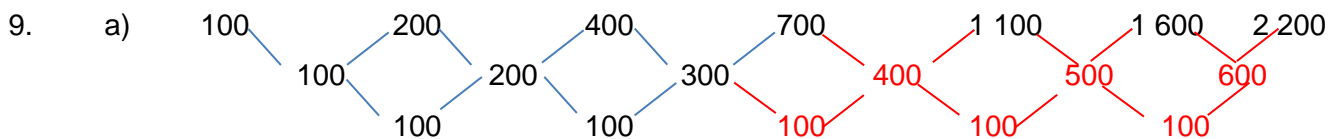
$$y = 2(-0.95) - 3$$

$$\text{and } y = 2(3.35) - 3$$

$$y = -4.9$$

$$y = 3.7$$

$$\therefore (-0.95; -4.9) \text{ and } (3.35; 3.7)$$



Therefore after 7 years, she will have saved:

$$100 + 200 + 400 + 700 + 1\,100 + 1\,600 + 2\,200 = 6\,300$$

b) $a + b + c = 100$

$$a = 50$$

$$3a + b = 100$$

$$b = 100 - 3(50) = -50$$

$$2a = 100$$

$$c = 100 - 50 - (-50) = 100$$

$$\therefore T_n = 50n^2 - 50n + 100$$

$$\therefore T_{21} = 50(21)^2 - 50(21) + 100$$

$$\therefore T_{21} = 21\,100$$

Irene will save R21 100 in the 21st year if this pattern continues.

