

SHARP

Worksheet 6 Memo – Equations and Inequalities

Grade 10 – Mathematics

1. a) $-2x + 6 = 18$
 $-2x = 18 - 6$
 $-2x = 12$
 $x = \frac{12}{-2}$
 $\therefore x = -6$
- b) $3(x - 1) = 6$
 $3x - 3 = 6$
 $3x = 6 + 3$
 $x = \frac{9}{3}$
 $\therefore x = 3$
- c) $3(x - 2) = 4(x + 5)$
 $3x - 6 = 4x + 20$
 $3x - 4x = 20 + 6$
 $-x = 26$
 $x = -26$
- d) $11x - \frac{1}{2} = -121\frac{1}{2}$
 $11x = -121\frac{1}{2} + \frac{1}{2}$
 $11x = -121$
 $x = -\frac{121}{11}$
 $x = -11$
- e) $\frac{x}{3} + 4 = \frac{x}{2} - 5$
 $2x + 24 = 3x - 30$
 $2x - 3x = -30 - 24$
 $-x = -54$
 $\therefore x = 54$
- f) $2(x - 3) + 3(2x + 8) = -3(x - 8)$
 $2x - 6 + 6x + 24 = -3x + 24$
 $8x + 3x = 24 - 24 + 6$
 $11x = 6$
 $\therefore x = \frac{6}{11}$
- g) $\frac{x+3}{5} = \frac{x+1}{2}$
 $2(x + 3) = 5(x + 1)$
 $2x + 6 = 5x + 5$
 $2x - 5x = 5 - 6$
 $-3x = -1$
 $x = \frac{-1}{-3}$
 $\therefore x = \frac{1}{3}$
- h) $\frac{3}{x} + 7 = 10$
 $3 + 7x = 10x$
 $7x - 10x = -3$
 $-3x = -3$
 $x = \frac{-3}{-3}$
 $\therefore x = 1$
- i) $-\frac{1}{4}x + \frac{3x}{5} = 2.1$
 $-5x + 12x = 42$
 $7x = 42$
 $x = \frac{42}{7}$
 $\therefore x = 6$
- j) $\frac{3}{4}(x - 2) = \frac{4}{3}(x + 7)$
 $\frac{3}{4}x - \frac{3}{2} = \frac{4}{3}x + \frac{28}{3}$
 $\frac{3}{4}x - \frac{4}{3}x = \frac{28}{3} + \frac{3}{2}$
 $\frac{9x - 16x}{12} = \frac{56 + 9}{6}$
 $-7x = 130$
 $\therefore x = -18\frac{4}{7}$

$$\begin{aligned} \text{k)} \quad & \frac{2}{x-2} + \frac{3}{x+2} = \frac{4}{x-2} \\ & 2(x+2) + 3(x-2) = 4(x+2) \\ & 2x+4+3x-6 = 4x+8 \\ & 5x-2 = 4x+8 \\ & 5x-4x = 8+2 \\ & \therefore x = 10 \end{aligned}$$

$$\begin{aligned} \text{l)} \quad & \frac{7}{(x-1)(x+3)} = \frac{4}{x+3} \\ & 7 = 4(x-1) \\ & 7 = 4x-4 \\ & 7+4 = 4x \\ & 11 = 4x \\ & \therefore x = \frac{11}{4} \\ & \therefore x = 2\frac{2}{3} \end{aligned}$$

$$\begin{aligned} \text{m)} \quad & \frac{3x+4}{(x-2)(x+2)} = \frac{7}{x-2} \\ & 3x+4 = 7(x+2) \\ & 3x+4 = 7x+14 \\ & 3x-7x = 14-4 \\ & -4x = 10 \\ & x = \frac{10}{-4} \\ & \therefore x = -2\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{n)} \quad & \frac{3-2x}{(x-1)(3x-16)} + \frac{5}{x-1} = 0 \\ & 3-2x+5(3x-16) = 0 \\ & 3-2x+15x-80 = 0 \\ & 13x = 77 \\ & x = \frac{77}{13} \\ & \therefore x = 5\frac{12}{13} \end{aligned}$$

$$\begin{aligned} \text{o)} \quad & \frac{3(x-4)(x+2)}{(x-4)(x-2)} = \frac{4}{x-2} \\ & \frac{3(x+2)}{x-2} = \frac{4}{x-2} \\ & 3(x+2) = 4 \\ & 3x+6 = 4 \\ & 3x = 4-6 \\ & 3x = -2 \\ & \therefore x = -\frac{2}{3} \end{aligned}$$

$$\begin{aligned} 2. \quad \text{a)} \quad & V = \pi r^2 h \quad \text{for } h \\ & \therefore \frac{V}{\pi r^2} = h \end{aligned}$$

$$\begin{aligned} \text{b)} \quad & V = \pi r^2 h \quad \text{for } r \\ & \frac{V}{\pi h} = r^2 \\ & \sqrt{\frac{V}{\pi h}} = r \end{aligned}$$

$$\begin{aligned} \text{c)} \quad & A = P(1+in) \quad \text{for } i \\ & \frac{A}{P} = 1+in \\ & \frac{A}{P} - 1 = in \\ & \therefore \frac{1}{n} \left(\frac{A}{P} - 1 \right) = i \end{aligned}$$

$$\begin{aligned} \text{d)} \quad & A = P(1+i)^n \quad \text{for } i \\ & \frac{A}{P} = (1+i)^n \\ & \sqrt[n]{\frac{A}{P}} = 1+i \\ & \therefore \sqrt[n]{\frac{A}{P}} - 1 = i \end{aligned}$$

$$\begin{aligned} \text{e)} \quad & 3 = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{for } c \\ & 6a = -b + \sqrt{b^2-4ac} \\ & 6a+b = \sqrt{b^2-4ac} \\ & (6a+b)^2 = b^2-4ac \\ & 36a^2+12ab+b^2 = b^2-4ac \\ & 36a^2+12ab = -4ac \\ & \frac{36a^2}{-4a} + \frac{12ab}{-4a} = c \rightarrow \therefore -9a-3b = c \end{aligned}$$

$$\begin{aligned} \text{f)} \quad & s = ut + \frac{1}{2}at^2 \quad \text{for } a \\ & s-ut = \frac{1}{2}at^2 \\ & 2(s-ut) = at^2 \\ & \frac{2}{t^2}(s-ut) = a \\ & \therefore \frac{2s}{t^2} - \frac{2u}{t} = a \end{aligned}$$

$$\begin{aligned} \text{g)} \quad s &= ut + \frac{1}{2}at^2 \quad \text{for } u \\ s - \frac{1}{2}at^2 &= ut \\ \frac{1}{t}\left(s - \frac{1}{2}at^2\right) &= u \\ \therefore \frac{s}{t} - \frac{at}{2} &= u \end{aligned}$$

$$\begin{aligned} \text{h)} \quad SA &= 2(lb + lh + bh) \quad \text{for } h \\ \frac{SA}{2} &= lb + lh + bh \\ \frac{SA}{2} - lb &= h(l + b) \\ \therefore \frac{\frac{SA}{2} - lb}{l + b} &= h \end{aligned}$$

$$\begin{aligned} \text{i)} \quad 0 &= ax^2 + bx + c \quad \text{for } a \\ -bx - c &= ax^2 \\ a &= \frac{-bx - c}{x^2} \\ \therefore a &= -\frac{b}{x} - \frac{c}{x^2} \end{aligned}$$

$$\begin{aligned} \text{j)} \quad T_n &= a(r)^{n-1} \quad \text{for } r \\ \frac{T_n}{a} &= (r)^{n-1} \\ \therefore r &= \sqrt[n-1]{\frac{T_n}{a}} \end{aligned}$$

$$\begin{aligned} 3. \quad \text{a)} \quad (4x - 1)(x - 3) &= -7 \\ 4x^2 - 12x - x + 3 + 7 &= 0 \\ 4x^2 - 13x + 10 &= 0 \\ (4x - 5)(x - 2) &= 0 \\ \therefore 4x = 5 \quad \text{OR} \quad x = 2 \\ \therefore x &= \frac{5}{4} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad x^2 - x - 12 &= 0 \\ (x - 4)(x + 3) &= 0 \\ \therefore x = 4 \quad \text{OR} \quad x = -3 \end{aligned}$$

$$\begin{aligned} \text{c)} \quad (3x - 5)(x + 2) &= 0 \\ \therefore 3x = 5 \quad \text{OR} \quad x = -2 \\ \therefore x &= \frac{5}{3} \end{aligned}$$

$$\begin{aligned} \text{d)} \quad x^2 - 4x &= -4x + 9 \\ x^2 - 4x + 4x - 9 &= 0 \\ x^2 - 9 &= 0 \end{aligned}$$

$$\begin{aligned} (x - 3)(x + 3) &= 0 \\ \therefore x = 3 \quad \text{OR} \quad x = -3 \end{aligned}$$

$$\begin{aligned} \text{e)} \quad (x - 7)(x + 5) &= -11 \\ x^2 + 5x - 7x - 35 + 11 &= 0 \\ x^2 - 2x - 24 &= 0 \\ (x - 6)(x + 4) &= 0 \\ \therefore x = 6 \quad \text{OR} \quad x = -4 \end{aligned}$$

$$\begin{aligned} \text{f)} \quad x^2 - 8x + 15 &= 0 \\ (x - 3)(x - 5) &= 0 \\ \therefore x = 3 \quad \text{OR} \quad x = 5 \end{aligned}$$

$$\begin{aligned} \text{g)} \quad x^2 + 13x + 40 &= 0 \\ (x + 5)(x + 8) &= 0 \\ \therefore x = -5 \quad \text{OR} \quad x = -8 \end{aligned}$$

$$\begin{aligned} \text{h)} \quad (x - 4)(x + 3) - 8 &= 0 \\ x^2 + 3x - 4x - 12 - 8 &= 0 \\ x^2 - x - 20 &= 0 \end{aligned}$$

$$\begin{aligned} (x - 5)(x + 4) &= 0 \\ \therefore x = 5 \quad \text{OR} \quad x = -4 \end{aligned}$$

$$\begin{aligned} \text{i)} \quad 4\left(x^2 - 2x + \frac{1}{2}\right) &= x \\ 4x^2 - 8x - 2 - x &= 0 \\ 4x^2 - 9x + 2 &= 0 \\ (4x - 1)(x - 2) &= 0 \\ \therefore 4x = 1 \quad \text{Or} \quad x = 2 \\ \therefore x &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{j)} \quad (2x + 1)(x - 2) - 7 &= 0 \\ 2x^2 - 4x + x - 2 - 7 &= 0 \\ 2x^2 - 3x - 9 &= 0 \\ (2x + 3)(x - 3) &= 0 \\ \therefore 2x = -3 \quad \text{OR} \quad x = 3 \\ \therefore x &= -\frac{3}{2} \end{aligned}$$

$$\begin{aligned}
 \text{k)} \quad & 4x^2 - 8x - 32 = 0 \\
 & 4(x^2 - 2x - 8) = 0 \\
 & x^2 - 2x - 8 = 0 \\
 & (x - 4)(x + 2) = 0 \\
 & \therefore x = 4 \quad \text{OR} \quad x = -2
 \end{aligned}$$

$$\begin{aligned}
 \text{l)} \quad & 3x^2 - 14x - 5 = 0 \\
 & (3x + 1)(x - 5) = 0 \\
 & \therefore 3x = -1 \quad \text{OR} \quad x = 5 \\
 & \therefore x = -\frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{m)} \quad & (3x - 8)(x + 2) = -8 \\
 & 3x^2 + 6x - 8x - 16 + 8 = 0 \\
 & 3x^2 - 2x - 8 = 0 \\
 & (3x + 4)(x - 2) = 0 \\
 & \therefore 3x = -4 \quad \text{OR} \quad x = 2 \\
 & \therefore x = -\frac{4}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{n)} \quad & 4x^2 - 8x - 5 = 0 \\
 & (2x + 1)(2x - 5) = 0 \\
 & \therefore 2x = -1 \quad \text{OR} \quad 2x = 5 \\
 & \therefore x = -\frac{1}{2} \qquad \qquad x = \frac{5}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{o)} \quad & x(x + 7) + 10 = 0 \\
 & x^2 + 7x + 10 = 0 \\
 & (x + 5)(x + 2) = 0 \\
 & \therefore x = -5 \quad \text{OR} \quad x = -2
 \end{aligned}$$

$$4. \quad \text{a)} \quad y = 3x + 4 \quad \dots 1 \qquad \text{and} \qquad y = -3x - 4 \quad \dots 2$$

Subs 1 into 2

$$\begin{aligned}
 & \therefore 3x + 4 = -3x - 4 \\
 & \therefore 3x + 3x = -4 - 4 \\
 & \therefore 6x = -8 \\
 & \therefore x = -\frac{8}{6} = -\frac{4}{3}
 \end{aligned}$$

Subs back into 1:

$$\begin{aligned}
 & \therefore y = 3\left(-\frac{4}{3}\right) + 4 \\
 & \therefore y = -4 + 4 \\
 & \therefore y = 0 \qquad \qquad \qquad \therefore \left(-\frac{4}{3}; 0\right)
 \end{aligned}$$

$$\text{b)} \quad y = \frac{1}{4}x + 2 \quad \dots 1 \quad \text{and} \quad y = 2x - 7 \quad \dots 2$$

Subs 1 into 2:

$$\begin{aligned}
 & \therefore \frac{1}{4}x + 2 = 2x - 7 \\
 & \therefore \frac{1}{4}x - 2x = -7 - 2 \\
 & \therefore -\frac{7}{4}x = -9 \\
 & \therefore -7x = -36 \\
 & \therefore x = \frac{36}{7} = 5\frac{1}{7}
 \end{aligned}$$

Subs back into 2:

$$\begin{aligned}
 & \therefore y = 2\left(5\frac{1}{7}\right) - 7 \\
 & \therefore y = 3\frac{2}{7} \qquad \qquad \qquad \therefore \left(5\frac{1}{7}; 3\frac{2}{7}\right)
 \end{aligned}$$

c) $y = x \dots 1$ and $y = \frac{2}{3}x - 1 \dots 2$

Subs 1 into 2

$$\therefore x = \frac{2}{3}x - 1$$

$$\therefore x - \frac{2}{3}x = -1$$

$$\therefore \frac{1}{3}x = -1$$

$$\therefore x = -3$$

Subs back into 1:

$$\therefore y = -3$$

$$\therefore (-3; -3)$$

d) $2y = -x + 2 \dots 1$ and $y - x = 4$
 $y = 4 + x \dots 2$

Subs 2 into 1

$$\therefore 2(4 + x) = -x + 2$$

$$\therefore 8 + 2x = -x + 2$$

$$\therefore 2x + x = 2 - 8$$

$$\therefore 3x = -6$$

$$\therefore x = -2$$

Subs back into 2:

$$\therefore y = 4 - 2$$

$$\therefore y = 2$$

$$\therefore (-2; 2)$$

e) $x - 3y = 4 \dots 1$ and $x = 7y + 3 \dots 2$

Subs 2 into 1

$$\therefore 7y + 3 - 3y = 4$$

$$\therefore 4y = 4 - 3$$

$$\therefore 4y = 1$$

$$\therefore y = \frac{1}{4}$$

Subs back into 2:

$$\therefore x = 7\left(\frac{1}{4}\right) + 3$$

$$\therefore x = 4\frac{3}{4}$$

$$\therefore \left(4\frac{3}{4}; \frac{1}{4}\right)$$

f) $2x = \frac{1}{3}y - 8 \dots 1$ and $y = x + 9 \dots 2$

Subs 2 into 1

$$\therefore 2x = \frac{1}{3}(x + 9) - 8$$

$$\therefore 2x = \frac{1}{3}x + 3 - 8$$

$$\therefore 2x - \frac{1}{3}x = -5$$

$$\therefore \frac{5}{3}x = -5$$

$$\therefore 5x = -15$$

$$\therefore x = -3$$

Subs back into 2:

$$\therefore y = -3 + 9$$

$$\therefore y = 6$$

$$\therefore (-3; 6)$$

g) $-3x + 6 = 2y \dots 1$ and

$y - x = 4$

$y = 4 + x \dots 2$

Subs 2 into 1

$\therefore -3x + 6 = 2(4 + x)$

$\therefore -3x + 6 = 8 + 2x$

$\therefore -3x - 2x = 8 - 6$

$\therefore -5x = 2$

$\therefore x = -\frac{2}{5}$

Subs back into 2:

$\therefore y = 4 - \frac{2}{5}$

$\therefore y = 3\frac{3}{5}$

$\therefore \left(-\frac{2}{5}; 3\frac{3}{5}\right)$

h) $x - 3y - 5 = 0$ and

$2x + 3y + 8 = 0 \dots 2$

$x = 3y + 5 \dots 1$

Subs 1 into 2

$\therefore 2(3y + 5) + 3y + 8 = 0$

$\therefore 6y + 10 + 3y + 8 = 0$

$\therefore 9y = -18$

$\therefore y = -2$

Subs back into 1:

$\therefore x = 3(-2) + 5$

$\therefore x = -1$

$\therefore (-1; -2)$

i) $y - \frac{3}{5}x + 9 = 0$ and

$\frac{1}{3}y = \frac{2}{3}x - 3 \dots 2$

$y = \frac{3}{5}x - 9 \dots 1$

Subs 1 into 2

$\therefore \frac{1}{3}\left(\frac{3}{5}x - 9\right) = \frac{2}{3}x - 3$

$\therefore \frac{1}{5}x - 3 = \frac{2}{3}x - 3$

$\therefore \frac{1}{5}x - \frac{2}{3}x = -3 + 3$

$\therefore \frac{7}{15}x = 0$

$\therefore x = 0$

Subs back into 1:

$\therefore y = \frac{3}{5}(0) - 9$

$\therefore y = -9$

$\therefore (0; -9)$

j) $5y - 3 = \frac{1}{2}x$ and

$9y = 4x - 7 \dots 2$

$10y - 6 = x \dots 1$

Subs 1 into 2

$\therefore 9y = 4(10y - 6) - 7$

$\therefore 9y = 40y - 24 - 7$

$\therefore 9y - 40y = -31$

$\therefore -31y = -31$

$\therefore y = 1$

Subs back into 1:

$\therefore x = 10(1) - 6$

$\therefore x = 4$

$\therefore (4; 1)$

$$5. \quad a) \quad 3x + 4 < 5$$

$$3x < 5 - 4$$

$$3x < 1$$

$$x < \frac{1}{3}$$

$$c) \quad 2(x - 6) > 0$$

$$2x - 12 > 0$$

$$2x > 12$$

$$x > 6$$

$$e) \quad \frac{x}{3} \geq \frac{x-1}{4}$$

$$4x \geq 3(x-1)$$

$$4x \geq 3x - 3$$

$$4x - 3x \geq -3$$

$$x \geq -3$$

$$g) \quad 3(x-5) + 4(x-2) \leq -2$$

$$3x - 15 + 4x - 8 \leq -2$$

$$7x - 23 \leq -2$$

$$7x \leq 21$$

$$x \leq 3$$

$$i) \quad 3(x+6) < 2\frac{2}{3}$$

$$3x + 18 < 2\frac{2}{3}$$

$$3x < 2\frac{2}{3} - 18$$

$$3x < -15\frac{1}{3}$$

$$x < -5\frac{1}{9}$$

$$b) \quad x - \frac{1}{2} \geq 3$$

$$x \geq 3 + \frac{1}{2}$$

$$x \geq 3\frac{1}{2}$$

$$d) \quad 5x + 4 \leq 3(x-1)$$

$$5x + 4 \leq 3x - 3$$

$$5x - 3x \leq -3 - 4$$

$$2x \leq -7$$

$$x \leq -\frac{7}{2}$$

$$f) \quad \frac{x+2}{5} - \frac{x-3}{2} \geq 0$$

$$2(x+2) - 5(x-3) \geq 0$$

$$2x + 4 - 5x + 15 \geq 0$$

$$-3x + 19 \geq 0$$

$$-3x \geq -19$$

$$x \leq \frac{19}{3}$$

$$x \leq 6\frac{1}{3}$$

$$h) \quad \frac{x-1}{4} - 9 > \frac{x+3}{2}$$

$$x - 1 - 36 > 2(x+3)$$

$$x - 37 > 2x + 6$$

$$x - 2x > 6 + 37$$

$$-x > 43$$

$$j) \quad \frac{x+1}{5} - \frac{2-x}{3} \leq \frac{3+x}{2}$$

$$6(x+1) - 10(2-x) \leq 15(3+x)$$

$$6x + 6 - 20 + 10x \leq 45 + 15x$$

$$16x - 14 \leq 45 + 15x$$

$$16x - 15x \leq 45 + 14$$

$$x \leq 59$$

$$6. \quad \text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$116 = \frac{507+507}{x+x-1}$$

$$116 = \frac{1014}{2x-1}$$

$$116(2x-1) = 1014$$

$$232x - 116 = 1014$$

$$232x = 1130$$

$$\therefore x = 4,87 \approx 5 \text{ hours}$$

\therefore It takes George 5 hours to get to Cape town and 4 hours to go back to Beaufort West.

7. let x = chocolate milkshakes and y = toasted sandwich

$$\therefore 3x + 2y = 104 \quad \text{and} \quad 4y + 5x = 190 \quad \dots 2$$

$$\therefore 2y = 104 - 3x$$

$$\therefore y = 52 - \frac{3}{2}x \quad \dots 1$$

Subs 1 into 2:

$$\therefore 4\left(52 - \frac{3}{2}x\right) + 5x = 190$$

$$\therefore 208 - 6x + 5x = 190$$

$$\therefore -x = 190 - 208$$

$$\therefore x = 18$$

Subs back into 1:

$$\therefore y = 52 - \frac{3}{2}(18)$$

$$\therefore y = 25$$

\therefore The chocolate milkshake costs R18 and the
the toasted sandwich costs R25.

8. $P = 2l + 2b$

$$\therefore 140 = 2(x) + 2(x + 4)$$

$$\therefore 140 = 2x + 2x + 8$$

$$\therefore 140 - 8 = 4x$$

$$\therefore 132 = 4x$$

$$\therefore 33m = x$$

$$\therefore \text{breadth} = 33m + 4m$$

$$= 37m$$

9. $Circumference = 2\pi r$

$$\therefore 31.416 = 2\pi \times r$$

$$\therefore \frac{31.416}{2\pi} = r$$

$$\therefore r = 5cm$$