

SHARP

Worksheet 11 – Patterns, Sequences and Series

Quadratic Patterns

1. Given the pattern: 62; 68; 72; 74; 74...
 - a) Determine the next three terms
 - b) Determine the general term, T_n , for the above sequence.
 - c) Find the 25th term of the series.
 - d) Prove that there is a term equal to 2.

2. Given the pattern: 59; 68; 82; 101...
 - a) Determine the next 3 terms of the pattern.
 - b) Determine T_n for the above pattern.
 - c) Find the term that is equal to 640.
 - d) Determine the 10th term.

3. Given the pattern: 6; 12; 21; 33; 48...
 - a) Determine the next three terms in the sequence.
 - b) Hence, or otherwise, determine the general term T_n , of the sequence.
 - c) Prove that every term in the above sequence is perfectly divisible by 3.
 - d) What is the 19th term?
 - e) Which term is equal to 201?

4. Given the sequence: a ; 21; 29; b ; 36...
 - a) Determine the values of a and b .
 - b) Hence, or otherwise, determine the general term, T_n of the sequence.
 - c) Find the 17th term.
 - d) Which term in the pattern is equal to 1?

5. Given the sequence: $a; -2; b; -2; 4\dots$
- Determine the values of a and b .
 - Hence, or otherwise, determine the general term, T_n , for the above pattern.
 - Determine which term is equal 68.
 - What are the two consecutive odd numbered terms that when added together give the value of 320?
6. On a certain long road with many robots, the municipality switches the robots to red and green in the following pattern:
 1 Red; 2 Green; 1 Red; 3 Green; 1 Red; 5 Green;
 1 Red; 8 Green and so on...
- If the pattern continues, determine how many green lights will be on in the 11th section of green lights?
 - If the average distance between the robots is 150m, determine how far you would have driven if you started at the first robot and finished after the 23rd green robot before the next red robot.

Arithmetic Sequences and Series

7. Given the series: $10 + 18 + 26 + 34 + \dots$
- Determine the first term, a , and the difference, d , of the above series.
 - Determine whether 100 is a part of the series.
 - Determine the 17th term.
 - Determine the sum of the first 10 terms.
 - Determine the sum of the values between term 11 and term 20.
8. Given the series: $7\frac{1}{3} + 9 + 10\frac{2}{3} + 12\frac{1}{3} + \dots$
- Determine the general term for the above series.
 - Determine the 14th term.
 - Which term is equal to $22\frac{1}{3}$?
 - Determine the sum of the first 20 terms.
 - Determine n if $S_n = 590\frac{1}{3}$

9. *Given that the following sequence is an arithmetic series:*
 $p - 1; \quad p + 2; \quad 2p \dots$
- Determine the value of p .
 - Hence, or otherwise, determine the first term and the common difference.
 - Find the 11th term.
 - Find the sum of the first 21 terms.
 - Find n if $S_n = 286$.
10. *Given that the following sequence is an arithmetic series:*
 $3 - 2p; \quad p + 3; \quad p - 3 \dots$
- Determine the value of p and hence the first three terms of the sequence.
 - Hence, or otherwise, determine the general term, T_n , in terms of n .
 - Find the 15th term.
 - Find the sum of the first 16 terms.
11. *Given that an arithmetic sequence has a 6th term equal to 39 and a 10th term equal to 83, determine:*
- The first term and the common difference.
 - The 9th term.
 - The sum of the first 19 terms.
 - The term which is equal to 127
 - n when $S_n = 4\,305$.
12. *Given that an arithmetic sequence has a fourth term equal to 7 and a ninth term equal to 57, determine:*
- the first term and the common difference
 - the terms between the fourth and ninth terms.
 - The sum of the first 18 terms.
 - The sum of the terms between term 9 and term 18.
 - n if $S_n = 384$

13. Given that the arithmetic sequence with the 6th term equal to 27 and the sum of the first 12 terms is equal to 504, determine:
- Determine the first term and the common difference for the series.
 - The 8th term.
 - The sum of the first 17 terms.
 - n if $T_n = 357$.

Geometric Sequences and Series

14. Given the geometric series: $6 + 30 + 150 + 750 + \dots$
- Determine the general term for the above series.
 - Determine the sum of the first 8 terms.
 - Is the series converging or diverging? Give a reason for your answer.
15. Given the following geometric series: $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots$
- Determine the general term, T_n , in terms of n .
 - Determine the sum of the first 12 terms.
 - Determine the sum of the first 50 terms.
 - Is the series converging or diverging? Give a reason for your answer.
 - Determine the sum to infinity.
 - Find n if $T_n = \frac{1}{16384}$.
16. Given the following geometric series: $14 + 21 + 31\frac{1}{2} + 47\frac{1}{4} + \dots$
- Determine the general term T_n in terms of n .
 - Determine the 8th term in the series.
 - Determine the sum of the first 16 terms.
 - Is this series a converging or diverging series? Give a reason for your answer.
 - Determine n if $S_n = 450\frac{13}{32}$.

17. Given that the following sequence is geometric:

$$p - 1; \quad \frac{1}{p-1}; \quad \frac{1}{6p+3} \dots$$

- Determine the value of p , and hence the first term and the common ratio.
- Hence, determine the first three terms.
- Determine the 7th term.
- Determine the sum of the first 10 terms.
- Which term is equal to $\frac{1}{19\,683}$?
- Determine the sum to infinity.

18. Given that the following series is geometric:

$$p - 4; \quad 2p + 2; \quad 9p - 1; \dots$$

- Determine the value of p if $p \neq 0$, and hence the values of the first three terms.
- Determine the sum of the first 10 terms.
- Which term is equal to 327 680?
- Determine the value of n if $S_n = 1\,705$.

19. In a certain geometric series the third term is $\frac{7}{2}$ and the 6th term is $\frac{189}{16}$.

Determine:

- Determine the first term and the common ratio.
- Determine the 8th term.
- Determine the sum of the first 11 terms.
- Is this a converging or diverging series? Give a reason for your answer.
- Determine n if $S_n = \frac{14413}{288}$

20. In a certain geometric series the 5th term is equal to $\frac{625}{486}$ and the 4th term is equal to $\frac{125}{81}$. Determine:

- Determine the first term and the common ratio.
- Determine the 7th term
- Determine the sum to infinity if possible.
- Find n if $S_n = 13,4159$.

Mixed Questions

21. Given the following sequence which is both arithmetic and geometric:
 $5; 2x; y \dots$ Determine the values for x and y .
22. Given that the first term of an arithmetic series is equal to the first term of a geometric series which is 2, that the 5th term of the arithmetic series is equal to the second term of the geometric series, and that the 8th term of the arithmetic series is equal to the 3rd term of the geometric series, determine the first three terms of both the arithmetic and geometric series.
23. Pruvén is doodling on the back of his exam pad and comes up with the following pattern:



Diagram 1

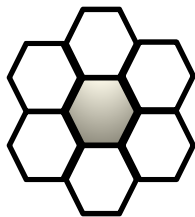


Diagram 2

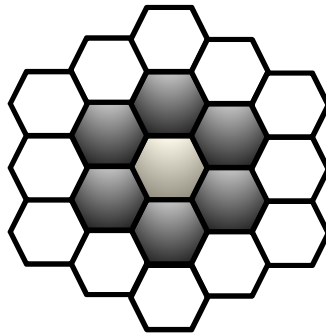


Diagram 3

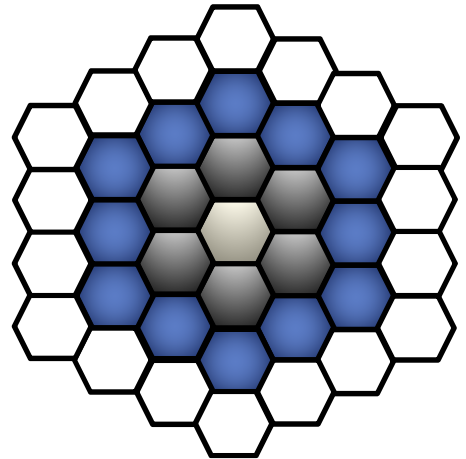


Diagram 4

- a) Determine whether the pattern is quadratic, arithmetic or geometric.
- b) Hence, or otherwise, determine the general term T_n in terms of n , for the doodles.
- c) Every hexagon Pruvén draws has a height of 1cm. If Pruvén's page has height of 15cm, how many hexagons would be in the doodle?