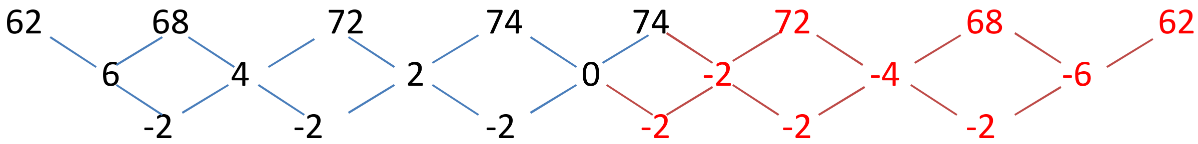
Worksheet 11 Memo – Patterns, Sequences and Series

Quadratic Patterns

1. *Given the pattern:* 62; 68; 72; 74; 74…

a)



$\therefore$ *next three terms are 72, 68 and 62*

b) $a + b + c = 62$ OR $T_n = a + (n-1)f + \frac{(n-1)(n-2)s}{2}$

$3a + b = 6$ $\therefore T_n = 62 + (n-1)6 + \frac{(n^2-3n+2)(-2)}{2}$

$2a = -2$ $\therefore T_n = 62 + 6n - 6 - n^2 + 3n - 2$

$\therefore a = -1$ $\therefore T_n = -n^2 + 9n + 54$

$3(-1) + b = 6$

$\therefore b = 9$

$-1 + 9 + c = 62$

$\therefore c = 54$ $\quad$ *and* $\therefore T_n = -n^2 + 9n + 54$

c) $T_{25} = -(25)^2 + 9(25) + 54$

$\therefore T_{25} = -346$

d) $2 = -n^2 + 9n + 54$

$\therefore 0 = -n^2 + 9n + 52$

$\therefore 0 = n^2 - 9n - 52$

$\therefore 0 = (n+4)(n-13)$

$n \neq -4 \quad or \quad n = 13$

2. *Given the pattern:* 59; 68; 82; 101…

a)

59 68 82 101 125 154 188

9 14 19 24 29 34

5 5 5 5 5

$\therefore$ *the next three terms are 125, 154 and 188.*

b) $a + b + c = 59$ OR $T_n = a + (n-1)f + \frac{(n-1)(n-2)s}{2}$

$3a + b = 9$ $\therefore T_n = 59 + (n-1)(9) + \frac{(n^2-3n+2)(5)}{2}$

$2a = 5$ $\therefore T_n = 59 + 9n - 9 + \frac{5}{2}n^2 - 7\frac{1}{2}n + 5$

$\therefore a = \frac{5}{2}\ or\ 2\frac{1}{2}$ $\therefore T_n = \frac{5}{2}n^2 + \frac{3}{2}n + 55$

$3\left(2\frac{1}{2}\right) + b = 9$

$\therefore b = \frac{3}{2}\ \ or\ \ 1\frac{1}{2}$

$2\frac{1}{2} + 1\frac{1}{2} + c = 59$

$\therefore c = 55$ $\therefore T_n = \frac{5}{2}n^2 + \frac{3}{2}n + 55$

c) $640 = \frac{5}{2}n^2 + \frac{3}{2}n + 55$

$\therefore 1280 = 5n^2 + 3n + 110$

$\therefore 0 = 5n^2 + 3n - 1170$

$\therefore 0 = (5n + 78)(n - 15)$

$\therefore n \neq -\frac{78}{5}$ or $n = 15$ ✓

d) $T_{10} = 2\frac{1}{2}(10)^2 + 1\frac{1}{2}(10) + 55$

$\therefore T_{10} = 320$

3. *Given the pattern:* 6; 12; 21; 33; 48…

a)

6 12 21 33 48 66 87 111

6 9 12 15 18 21 24

3 3 3 3 3 3

b) $a + b + c = 6$

$3a + b = 6$

$2a = 3$

$\therefore a = \frac{3}{2} \ or\ 1\frac{1}{2}$

$3\left(1\frac{1}{2}\right) + b = 6$

$\therefore b = 1\frac{1}{2} \ or\ \frac{3}{2}$

$1\frac{1}{2} + 1\frac{1}{2} + c = 6$

$\therefore c = 3$ $\quad\quad \therefore T_n = 1\frac{1}{2}n^2 + 1\frac{1}{2}n + 3$

OR

$T_n = a + (n-1)f + \frac{(n-1)(n-2)s}{2}$

$\therefore T_n = 6 + (n-1)(6) + \frac{(n^2-3n+2)(3)}{2}$

$\therefore T_n = 6 + 6n - 6 + \frac{3}{2}n^2 - 4\frac{1}{2}n + 3$

$\therefore T_n = \frac{3}{2}n^2 + 1\frac{1}{2}n + 3$

c) $\frac{T_n}{3} = \frac{\frac{3}{2}n^2 + \frac{3}{2}n + 3}{3}$ $\quad$ OR $\quad Tn = 3\left(\frac{1}{2}n^2 + \frac{1}{2}n + 1\right)$

$\therefore = \frac{3}{2}n^2 \div 3 + \frac{3}{2}n \div 3 + 3 \div 3$

$\therefore = \frac{1}{2}n^2 + \frac{1}{2}n + 1$

$\therefore$ *every term is perfectly divisible by 3 because the general term is perfectly divisible by 3.*

d) $T_{19} = 1\frac{1}{2}(19)^2 + 1\frac{1}{2}(19) + 3$

$\therefore T_{19} = 573$

e) $201 = 1\frac{1}{2}n^2 + 1\frac{1}{2}n + 3$

$\therefore 402 = 3n^2 + 3n + 6$

$\therefore 0 = 3n^2 + 3n - 396$

$\therefore 0 = n^2 + n - 132$

$\therefore 0 = (n+12)(n-11)$

$\therefore n \neq -12 \quad or \quad n = 11$

4. *Given the sequence:* $\quad$ *a*; $\quad$ 21; $\quad$ 29; $\quad$ *b*; $\quad$ 36….

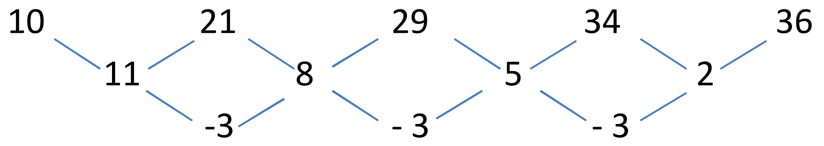
a)

a		21		29		*b*		36
	21 – *a*		8		*b* – 29		36 – *b*	
		-13 + *a*		*b* – 37		65 – 2*b*		

$\therefore b - 37 = 65 - 2b$
$\therefore 3b = 102$
$\therefore b = 34$

$\therefore \ -13 + a = b - 37$
$\therefore a = 34 - 37 + 13$
$\therefore a = 10$

b)



$a + b + c = 10$ OR $T_n = a + (n-1)f + \frac{(n-1)(n-2)s}{2}$

$3a + b = 11$ $\qquad \therefore T_n = 10 + (n-1)(11) + \frac{(n^2-3n+2)(-3)}{2}$

$2a = -3$ $\qquad \therefore Tn = 10 + 11n - 11 - \frac{3}{2}n^2 + \frac{9}{2}n - 3$

$\therefore a = -\frac{3}{2} \ or \ -1\frac{1}{2}$ $\qquad \therefore T_n = -\frac{3}{2}n^2 + 15\frac{1}{2}n - 4$

$3\left(-\frac{3}{2}\right) + b = 11$

$\therefore b = 15\frac{1}{2}$

$-\frac{3}{2} + 15\frac{1}{2} + c = 10$

$\therefore c = -4$

c) $T_{17} = -\frac{3}{2}(17)^2 + 15\frac{1}{2}(17) - 4$
$\therefore T_{17} = -174$

d) $1 = -\frac{3}{2}n^2 + 15\frac{1}{2}n - 4$
$\therefore 2 = -3n^2 + 31n - 8$
$\therefore 0 = -3n^2 + 31n - 10$
$\therefore 0 = 3n^2 - 31n + 10$
$\therefore 0 = (3n-1)(n-10)$
$\therefore n \neq \frac{1}{3} \quad or \quad n = 10$ ✓

5. *Given the sequence:* *a;* -2; *b;* -2; 4....

a)

a ; -2 ; *b* ; -2 ; 4

First differences: -2 − *a* ; *b* + 2 ; -2 − *b* ; 6

Second differences: *b* +2 +2 +*a* ; -2 − *b* −*b* − 2 ; 6 + 2 + *b*

$\therefore 8 + b = -4 - 2b$

$\therefore 12 = -3b$

$\therefore b = -4$

$\therefore b + 2 + 2 + a = -4 - 2b$

$\therefore b + 4 + a = -4 - 2b$

$\therefore a = -4 - 2(-4) - (-4) - 4$

$\therefore a = 4$

b) 4 ; - 2 ; -4 ; -2 ; 4

First differences: -6 ; -2 ; 2 ; 6

Second differences: 4 ; 4 ; 4

$a + b + c = 4$ OR $T_n = a + (n-1)f + \frac{(n-1)(n-2)s}{2}$

$3a + b = -6$ $\therefore T_n = 4 + (n-1)(-6) + \frac{(n^2-3n+2)(4)}{2}$

$2a = 4$ $\therefore T_n = 4 - 6n + 6 + 2n^2 - 6n + 4$

$\therefore a = 2$ $\therefore T_n = 2n^2 - 12n + 14$

$3(2) + b = -6$

$\therefore b = -12$

$2 - 12 + c = 4$

$\therefore c = 14$ $T_n = 2n^2 - 12n + 14$

c) $68 = 2n^2 - 12n + 14$

$\therefore 0 = 2n^2 - 12n - 54$

$\therefore 0 = n^2 - 6n - 27$

$\therefore 0 = (n+3)(n-9)$

$\therefore n \neq -3 \quad or\ n = 9$

d) $320 = 2(2n+1)^2 - 12(2n+1) + 14 + 2(2n+3)^2 - 12(2n+3) + 14$

$320 = 2(4n^2 + 4n + 1) - 24n - 12 + 14 + 2(4n^2 + 12n + 9) - 24n - 36 + 14$

$\therefore 320 = 8n^2 + 8n + 2 - 48n - 20 + 8n^2 + 24n + 18$

$\therefore 0 = 16n^2 - 16n - 320$

$\therefore 0 = n^2 - n - 20$

$\therefore 0 = (n-5)(n+4)$ $\quad\quad \therefore n = 5 \quad or\; n \neq -4$

$\therefore$ the 1st odd term is $= 2(5) + 1 = 11$

The 2nd odd term is $= 2(5) + 3 = 13$

$\therefore$ The 11th and 13th terms added together will give you 320.

6. a)

2   3   5   8

1   2   3

1   1

$a + b + c = 2$ OR $T_n = a + (n-1)f + \frac{(n-1)(n-2)s}{2}$

$3a + b = 1$ $\quad \therefore T_n = 2 + (n-1)(1) + \frac{(n^2-3n+2)(1)}{2}$

$2a = 1$ $\quad \therefore T_n = 2 + n - 1 + \frac{1}{2}n^2 - \frac{3}{2}n + 1$

$\therefore a = \frac{1}{2}$ $\quad \therefore T_n = \frac{1}{2}n^2 - \frac{1}{2}n + 2$

$3\left(\frac{1}{2}\right) + b = 1$

$\therefore b = -\frac{1}{2}$

$\frac{1}{2} - \frac{1}{2} + c = 2$

$\therefore c = 2$ $\quad\quad \therefore T_n = \frac{1}{2}n^2 - \frac{1}{2}n + 2$

b)

2   3   5   8   12   17   23

1   2   3   4   5   6

1   1   1   1   1

$\therefore 1 + 2 + 1 + 3 + 1 + 5 + 1 + 8 + 1 + 12 + 1 + 17 + 1 + 23$

$= 77\; robots \times 150m$

$= 11\,550m \quad or\; 11{,}55km$

Arithmetic Sequences and Series

7. *Given the series:* 10 + 18 + 26 + 34 + …

a) $a = 10$
$d = 8$

b) $100 = 10 + (n-1)(8)$
$90 = 8n - 8$
$98 = 8n$
$\therefore n = 12\frac{1}{4}$ $\therefore$ *100 is not a part of the series.*

c) $T_{17} = 10 + (17-1)(8)$
$T_{17} = 138$

d) $S_n = \frac{n}{2}[2a + (n-1)d]$
$\therefore S_{10} = \frac{10}{2}[2(10) + (9)(8)]$
$= 5[20 + 72]$
$= 5[92]$
$= 460$

e) $S_{20-10} = \frac{20}{2}[2(10) + (19)(8)] - 460$
$= 10[20 + 152] - 460$
$= 10[172] - 460$
$= 1720 - 460$
$= 1\ 260$

8. *Given the series:* $7\frac{1}{3} + 9 + 10\frac{2}{3} + 12\frac{1}{3} + \cdots.$

a) $a = 7\frac{1}{3}$ $\quad and\ d = 9 - 7\frac{1}{3} = \frac{5}{3}$
$\therefore T_n = 7\frac{1}{3} + (n-1)\left(\frac{5}{3}\right) = 5\frac{2}{3} + 1\frac{2}{3}n$

b) $T_{14} = 7\frac{1}{3} + (13)\left(\frac{5}{3}\right)$
$\therefore T_{14} = 7\frac{1}{3} + 21\frac{2}{3}$ $\therefore T_{14} = 29$

c) $22\frac{1}{3} = 7\frac{1}{3} + (n-1)\left(\frac{5}{3}\right)$

$\therefore 15 = \left(\frac{5}{3}\right)(n-1)$

$\therefore 9 = n - 1$

$\therefore n = 10$

d) $S_{20} = \frac{20}{2}\left[2\left(7\frac{1}{3}\right) + (19)\left(\frac{5}{3}\right)\right]$

$\therefore S_{20} = 10\left[14\frac{2}{3} + 31\frac{2}{3}\right]$

$\therefore S_{20} = 10\left[46\frac{1}{3}\right]$

$\therefore S_{20} = 463\frac{1}{3}$

e) $590\frac{1}{3} = \frac{n}{2}\left[2\left(7\frac{1}{3}\right) + (n-1)\left(\frac{5}{3}\right)\right]$

$\therefore 11\,80\frac{2}{3} = n\ \left[14\frac{2}{3} + \frac{5}{3}n - \frac{5}{3}\right]$

$\therefore 11\,80\frac{2}{3} = 14\frac{2}{3}n + \frac{5}{3}n^2 - \frac{5}{3}n$

$\therefore 0 = \frac{5}{3}n^2 + 13n - 1180\frac{2}{3}$

$\therefore 0 = n^2 + 39n - 3\,542$

$\therefore 0 = (5n + 154)(n - 23)$

$\therefore n \neq -\frac{154}{5} \quad or \quad n = 23$

9. *Given that the following sequence is an arithmetic series:*

$p - 1; \quad p + 2; \quad 2p\ldots$

a) $T_2 - T_1 = T_3 - T_2$

$\therefore p + 2 - (p - 1) = 2p - (p + 2)$

$\therefore p + 2 - p + 1 = 2p - p - 2$

$\therefore 3 = p - 2$

$\therefore p = 5$

b) 4 7 10…

$\therefore a = 4$

$\therefore d = 3$

c) $T_{11} = 4 + (10)(3)$

$T_{11} = 34$

d) $S_{21} = \frac{21}{2}[2(4) + (20)(3)]$

$\therefore S_{21} = \frac{21}{2}[8 + 60]$

$\therefore S_{21} = \frac{21}{2}[68]$

$\therefore S_{21} = 714$

e) $286 = \frac{n}{2}[2(4) + (n-1)(3)]$

$\therefore 572 = n[8 + 3n - 3]$

$\therefore 0 = 3n^2 + 5n - 572$

$\therefore 0 = (3n + 44)(n - 13)$

$\therefore n \neq -\frac{44}{3} \qquad n = 13$ ✓

10. *Given that the following sequence is an arithmetic series:*

$3 - 2p; \quad p + 3; \quad p - 3 \ldots$

a) $T_2 - T_1 = T_3 - T_2$

$\therefore p + 3 - (3 - 2p) = p - 3 - (p + 3)$

$\therefore p + 3 - 3 + 2p = p - 3 - p - 3$

$\therefore 3p = -6$

$\therefore p = -2$

$\therefore T_1 = 3 - 2(-2) = 7 \qquad \therefore T_2 = -2 + 3 = 1$

$\therefore T_3 = -2 - 3 = -5$

b) $T_n = 7 + (n-1)(-6) = 13 - 6n$

c) $T_{15} = 7 + (14)(-6)$

$\therefore T_{15} = -77$

d) $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{16} = \frac{16}{2}[2(7) + (15)(-6)]$

$\therefore S_{16} = 8[14 - 90]$

$\therefore S_{16} = 8[-76]$

$\therefore S_{16} = -608$

11. *Given that an arithmetic sequence has a 6th term equal to 39 and a 10th term equal to 83, determine:*

T_6 = 39 *T_{10} = 83*

a) $\therefore 39 = a + 5d$

$\therefore 83 = a + 9d$

$\therefore -44 = -4d$

$\therefore d = 11$

$\therefore 39 = a + 5(11)$

$\therefore a = -16$

b) $T_9 = -16 + (8)(11)$

$\therefore T_9 = 72$

c) $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{19} = \frac{19}{2}[2(-16) + (18)(11)]$

$\therefore S_{19} = \frac{19}{2}[-32 + 198]$

$\therefore S_{19} = \frac{19}{2}[166]$

$\therefore S_{19} = 1\,577$

d) $127 = -16 + (n-1)(11)$

$\therefore 143 = 11(n-1)$

$\therefore 13 = n - 1$

$\therefore n = 14$

e)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore 4\,305 = \frac{n}{2}[2(-16) + (n-1)(11)]$

$\therefore 8\,610 = n[-32 + 11n - 11]$

$\therefore 0 = 11n^2 - 43n - 8610$

$\therefore 0 = (11n + 287)(n - 30)$

$\therefore n \neq -\frac{287}{11} \quad or \quad n = 30$ ✓

12. *Given that an arithmetic sequence has a fourth term equal to 7 and a ninth term equal to 57, determine:*

$T_4 = 7$ $T_9 = 57$

a) $\therefore 7 = a + 3d$

$\therefore 57 = a + 8d$

$\therefore -50 = -5d$

$\therefore d = 10$ $\therefore 7 = a + 3(10)$

$\therefore a = -23$

b) 7; 17; 27; 37; 47; 57...

c) $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{18} = \frac{18}{2}[2(-23) + (17)(10)]$

$\therefore S_{18} = 9[-46 + 170]$

$\therefore S_{18} = 9[124]$

$\therefore S_{18} = 1\ 116$

d) $S_{18} - S_8 = 1116 - \frac{8}{2}[2(-23) + (7)(10)]$

$\therefore S_{18} - S_9 = 1\ 116 - 4[-46 + 70]$

$\therefore S_{18} - S_9 = 1\ 116 - 4[24]$

$\therefore S_{18} - S_9 = 1\ 116 - 96$

$\therefore S_{18} - S_9 = 1020$

e) $\therefore 384 = \frac{n}{2}[2(-23) + (n-1)(10)]$

$\therefore 768 = n[-46 + 10n - 10]$

$\therefore 0 = 10n^2 - 56n - 768$

$\therefore 0 = 5n^2 - 28n - 384$

$\therefore 0 = (5n + 32)(n - 12)$

$\therefore n \neq -\frac{32}{5} \quad or \quad n = 12$ ✓

13. *Given that the arithmetic sequence with the 6th term equal to 27 and the sum of the first 12 terms is equal to 504, determine:*

$T_6 = 27$ $S_{12} = 504$

a) $27 = a + 5d$

$\therefore a = 27 - 5d$

$\therefore 504 = \frac{12}{2}[2a + 11d]$ Substitute *a* in

$\therefore 504 = 6[2(27 - 5d) + 11d]$

$\therefore 84 = 54 - 10d + 11d$

$\therefore 30 = d$

$\therefore a = 27 - 5(30)$

$\therefore a = -123$

b) $T_8 = -123 + (7)(30)$

$\therefore T_8 = 87$

c) $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{17} = \frac{17}{2}[2(-123) + 16(30)]$

$\therefore S_{17} = \frac{17}{2}[-246 + 480]$

$\therefore S_{17} = \frac{17}{2}[234]$

$\therefore S_{17} = 1\,989$

d) $357 = -123 + (n-1)(30)$

$\therefore 480 = (30)(n-1)$

$\therefore 16 = n - 1$

$\therefore n = 17$

Geometric Sequences and Series

14. *Given the geometric series:* 6 + 30 + 150 + 750 + …

a) $a = 6$

$r = 5$ $\therefore T_n = 6.5^{n-1}$

b) $S_n = \frac{a(1-r^n)}{1-r}$

$\therefore S_8 = \frac{6(1-5^8)}{1-5}$

$\therefore S_8 = 585\,936$

c) *It is diverging because the ratio is greater than 1.*

15. *Given the following geometric series:* $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

a) $a = 1$

$r = \frac{1}{4}$ $\qquad \therefore T_n = 1.\left(\frac{1}{4}\right)^{n-1}$

b) $S_n = \frac{a(1-r^n)}{1-r}$

$\therefore S_{12} = \frac{1\left(1-\left(\frac{1}{4}\right)^{12}\right)}{1-\frac{1}{4}}$

$\therefore S_{12} = 1{,}33$

c) $\therefore S_{50} = \frac{1\left(1-\left(\frac{1}{4}\right)^{50}\right)}{1-\frac{1}{4}}$

$\therefore S_{50} = 1\frac{1}{3}$

d) Converging, $-1 < r < 1$

e) $S_\infty = \frac{a}{1-r}$

$\therefore S_\infty = \frac{1}{1-\frac{1}{4}}$

$\therefore S_\infty = 1\frac{1}{3}$

f) $T_n = \frac{1}{16\,384} = 1.\left(\frac{1}{4}\right)^{n-1}$

$\therefore \log_{\frac{1}{4}} \frac{1}{16\,384} = n - 1$

$\therefore n - 1 = 7$

$\therefore n = 8$

16. *Given the following geometric series:* $14 + 21 + 31\frac{1}{2} + 47\frac{1}{4} + \cdots$

a) $a = 14$

$r = \frac{21}{14} = \frac{3}{2}$ $\quad\quad \therefore T_n = 14.\left(\frac{3}{2}\right)^{n-1}$

b) $T_8 = 14.\left(\frac{3}{2}\right)^7$

$\therefore T_8 = 239\frac{13}{64}$

c) $S_n = \frac{a(1-r^n)}{1-r}$

$\therefore S_{16} = \frac{14\left(1-\left(\frac{3}{2}\right)^{16}\right)}{1-\frac{3}{2}}$

$\therefore S_{16} = 18\ 363,5434$

d) *It is diverging, r is greater than 1.*

e) $450\frac{13}{32} = \frac{a(1-r^n)}{1-r}$

$\therefore \frac{14\ 413}{32} = \frac{14\left(1-\left(\frac{3}{2}\right)^n\right)}{1-\frac{3}{2}}$

$\therefore -\frac{14\ 413}{64} = 14\left(1 - \left(\frac{3}{2}\right)^n\right)$

$\therefore -\frac{2059}{64} = 1 - \left(\frac{3}{2}\right)^n$

$\therefore -\frac{2187}{128} = -\left(\frac{3}{2}\right)^n$

$\therefore \log_{\frac{3}{2}}\left(\frac{2\ 187}{128}\right) = n$

$\therefore n = 7$

17. *Given that the following sequence is geometric:*

$p - 1; \quad \frac{1}{p-1}; \quad \frac{1}{6p+3} \ldots$

a)* $\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\therefore \frac{\frac{1}{p-1}}{p-1} = \frac{\frac{1}{6p+3}}{\frac{1}{p-1}}$

$\therefore \frac{1}{p-1} \times \frac{1}{p-1} = \frac{1}{6p+3} \times \frac{p-1}{1}$

$\therefore \frac{1}{(p-1)^2} = \frac{p-1}{6p+3}$

$\therefore \frac{6p+3}{(p-1)^2} = p - 1$

$\therefore 6p + 3 = (p-1)^3$

$\therefore 6p + 3 = p^3 - 3p^2 + 3p - 1$

$\therefore 0 = p^3 - 3p^2 - 3p - 4$

$\therefore 0 = (p-4)(p^2 + kp + 1)$

$\therefore -4p^2 + kp^2 = -3p^2$

$\therefore kp^2 = p^2$

$\therefore k = 1$

$\therefore 0 = (p-4)(p^2 + p + 1)$

$\therefore p = 4$ *can't factorise this trinomial*

b) $3; \quad \frac{1}{3}; \quad \frac{1}{27} \ldots$

c) $T_n = 3.\left(\frac{1}{9}\right)^{n-1}$

$\therefore T_7 = 3.\left(\frac{1}{9}\right)^6$

$\therefore T_7 = \frac{1}{177\,147}$

d) $S_{10} = \frac{3\left(1-\left(\frac{1}{9}\right)^{10}\right)}{1-\frac{1}{9}}$

$\therefore S_{10} = 3{,}37499\dot{9}$

e) $\frac{1}{19\,683} = 3.\left(\frac{1}{9}\right)^{n-1}$

$\therefore \frac{1}{59\,049} = \left(\frac{1}{9}\right)^{n-1}$

$\therefore \log_{\frac{1}{9}}\left(\frac{1}{59\,049}\right) = n - 1$

$\therefore n - 1 = 5$

$\therefore n = 6$

f) $S_\infty = \frac{a}{1-r}$

$\therefore S_\infty = \frac{3}{1-\frac{1}{9}}$

$\therefore S_\infty = 3\frac{3}{8}$

18. *Given that the following series is geometric:*

$p-4; \quad 2p+2; \quad 9p-1; \ldots$

a) $\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\therefore \frac{2p+2}{p-4} = \frac{9p-1}{2p+2}$

$\therefore (2p+2)^2 = (9p-1)(p-4)$

$\therefore 4p^2 + 8p + 4 = 9p^2 - 37p + 4$

$\therefore 0 = 5p^2 - 45p$

$\therefore 0 = 5p(p-9)$

$\therefore p = 0 \quad or \quad p = 9$ ✓

$\therefore T_1 = 5$ $\quad\therefore T_2 = 20$ $\quad$ and $\therefore T_3 = 80$

b) $S_n = \frac{a(1-r^n)}{1-r}$ $\quad a = 5; \quad r = 4$

$\therefore S_{10} = \frac{5(1-4^{10})}{1-4}$

$\therefore S_{10} = 1\,747\,625$

c) $327\,680 = 5.(4)^{n-1}$

$\therefore 65\,536 = 4^{n-1}$

$\therefore \log_4 65\,536 = n-1$

$\therefore n - 1 = 8$

$\therefore n = 9$

d) $1\,705 = \frac{5(1-4^n)}{1-4}$

$\therefore -5\,115 = 5(1-4^n)$

$\therefore -1\,023 = 1 - 4^n$

$\therefore -1024 = -4^n$

$\therefore \log_4 1024 = n$

$\therefore n = 5$

19. *In a certain geometric series the third term is* $\frac{7}{2}$ *and the* 6^{th} *term is* $\frac{189}{16}$.

Determine:

$T_3 = \frac{7}{2}$ $\quad\quad$ $T_6 = \frac{189}{16}$

a) $\therefore ar^5 = \frac{189}{16} \ldots \quad 1$

$\therefore ar^2 = \frac{7}{2} \ldots \quad 2$

$\therefore 1 \div 2$

$\therefore r^3 = \frac{27}{8}$

$\therefore r = \sqrt[3]{\frac{27}{8}}$

$\therefore r = \frac{3}{2}$

$\therefore ar^2 = \frac{7}{2}$

$\therefore a\left(\frac{3}{2}\right)^2 = \frac{7}{2}$

$\therefore a = \frac{14}{9}$

b) $T_n = \frac{14}{9}\left(\frac{3}{2}\right)^{n-1}$

$\therefore T_8 = \frac{14}{9}\left(\frac{3}{2}\right)^7$

$\therefore T_8 = 26\frac{37}{64} \;\; or \;\; \frac{1\,701}{64}$

c) $S_n = \frac{a(1-r^n)}{1-r}$

$\therefore S_{11} = \frac{\frac{14}{9}\left(1-\left(\frac{3}{2}\right)^{11}\right)}{1-\frac{3}{2}}$

$\therefore S_{11} = 265{,}9924 \ldots$

d) *Diverging; r is greater than 1.*

e) $\frac{14413}{288} = \frac{\frac{14}{9}\left(1-\left(\frac{3}{2}\right)^{n}\right)}{1-\frac{3}{2}}$

$\therefore -\frac{14413}{576} = \frac{14}{9}\left(1-\left(\frac{3}{2}\right)^n\right)$

$\therefore -\frac{2059}{128} = 1-\left(\frac{3}{2}\right)^n$

$\therefore -\frac{2187}{128} = -\left(\frac{3}{2}\right)^n$

$\therefore\ \log_{\left(\frac{3}{2}\right)}\left(\frac{2187}{128}\right) = n$
$\therefore n = 7$

20. *In a certain geometric series the 5^th term is equal to $\frac{625}{486}$ and the 4^th term is equal to $\frac{125}{81}$. Determine:*

$T_5 = \frac{625}{486}$ *and* $T_4 = \frac{125}{81}$

a) $ar^4 = \frac{625}{486} \ldots$ 1
$ar^3 = \frac{125}{81} \ldots$ 2
$\therefore 1 \div 2$
$\therefore r = \frac{5}{6}$

$\therefore a\left(\frac{5}{6}\right)^3 = \frac{125}{81}$
$\therefore a = \frac{8}{3}$

b) $T_n = \frac{8}{3}\left(\frac{5}{6}\right)^{n-1}$
$\therefore T_7 = \frac{8}{3}\left(\frac{5}{6}\right)^6$
$\therefore T_7 = 0{,}89306 \ldots$

c) $S_\infty = \frac{a}{1-r}$
$\therefore S_\infty = \frac{\frac{8}{3}}{1-\frac{5}{6}}$
$\therefore S_\infty = 16$ (Possible because -1 < r < 1).

d) $13{,}4159 = \frac{\frac{8}{3}\left(1-\left(\frac{5}{6}\right)^n\right)}{1-\frac{5}{6}}$
$\therefore 2{,}235983333 = \frac{8}{3}\left(1-\left(\frac{5}{6}\right)^n\right)$
$\therefore 0{,}83849375 = 1-\left(\frac{5}{6}\right)^n$
$\therefore\ -0{,}16150625 = -\left(\frac{5}{6}\right)^n$
$\therefore\ \log_{\frac{5}{6}}(0{,}16150625) = n$
$\therefore n = 9{,}999977$
$\therefore n = 10$

Mixed Questions

21. *Arithmetic*

a = 5

d = $2x - 5 = y - 2x$

$\therefore 4x - 5 = y \ldots \quad 1$

Geometric

a = 5

$r = \frac{2x}{5} = \frac{y}{2x}$

$\therefore 4x^2 = 5y \ldots \quad 2$

Substitute 1 into 2

$\therefore 4x^2 = 5(4x - 5)$

$\therefore 4x^2 = 20x - 25$

$\therefore 4x^2 - 20x + 25 = 0$

$\therefore (2x - 5)(2x - 5) = 0$

$\therefore x = \frac{5}{2}$ $\qquad \therefore y = 4\left(\frac{5}{2}\right) - 5$

$\therefore y = 5$

22. *a* = *a* = 2

$a + 4d = ar$ $\qquad a + 7d = ar^2$

$2 + 4d = 2r$ $\qquad 2 + 7d = 2r^2$

$1 + 2d = r$ $\qquad$ subs in → $2 + 7d = 2(1 + 2d)^2$

$\therefore 2 + 7d = 2(1 + 4d + 4d^2)$

$\therefore 2 + 7d = 2 + 8d + 8d^2$

$\therefore \quad 0 = 8d^2 + d$

$\therefore \quad 0 = d(8d + 1)$

$\therefore \quad d = 0 \quad or \quad d = -\frac{1}{8}$

$\therefore 1 + 2(0) = r$ $\qquad$ OR $\qquad 1 + 2\left(-\frac{1}{8}\right) = r$

$\therefore r = 1$ $\qquad \therefore r = \frac{3}{4}$

∴ arithmetic: 2; 2; 2;… OR 2; $1\frac{7}{8}$; $1\frac{3}{4}$ …

∴ geometric: 2; 2; 2; OR 2; $1\frac{1}{2}$; $1\frac{1}{8}$ ….

23. 1   7   19   37

6   12   18

6   6

a) *It is not linear (no first common difference).*
It is not geometric (no common ratio).
It IS quadratic (there is a second common difference).

b) $a + b + c = 1$ OR $T_n = a + (n-1)f + \frac{(n-1)(n-2)s}{2}$

$3a + b = 6$ $\therefore T_n = 1 + (n-1)(6) + \frac{(n^2-3n+2)(6)}{2}$

$2a = 6$ $\therefore T_n = 1 + 6n - 6 + 3n^2 - 9n + 6$

$\therefore a = 3$ $\therefore T_n = 3n^2 - 3n + 1$

$3(3) + b = 6$

$\therefore b = -3$

$3 - 3 + c = 1$

$\therefore c = 1$ $\therefore T_n = 3n^2 - 3n + 1$

c) 1 3 5 7 (number of hexagons tall)
2 2 2

$\therefore T_n = 1 + (n-1)(2) = 15$
$\therefore 2(n-1) = 14$
$\therefore n - 1 = 7$
$\therefore n = 8$

$\therefore T_8 = 3(8)^2 + 3(8) + 1$
$\therefore T_8 = 217$ hexagons in the doodle.