Worksheet 7 Memorandum: Euclidean Geometry

Grade 11 Mathematics

1. a)

$\hat{C}_{1+2} = 90°$	Radius ⊥ Tangent
$\hat{C}_{3+4} = 90°$	Radius ⊥ Tangent
$\therefore\ \hat{C}_{1+2} = \hat{C}_{3+4}$	Both = 90°
But $\hat{C}_2 = \hat{C}_3$	Given
$\therefore \hat{C}_1 = \hat{C}_4$	

b)

$\hat{B}_2 = \hat{C}_2$	AB = AC (radii), isosceles Δ	
$\hat{B}_1 = \hat{D}_1$	AB = AD (radii), isosceles Δ	
$\hat{C}_3 = \hat{D}_2$	AD = AC (radii), isosceles Δ	
$\hat{C}_2 = \hat{C}_3$	Given	
$\hat{B}_2 = \hat{D}_2$	$\hat{B}_2 = \hat{C}_2 = \hat{C}_3 = \hat{D}_2$	(use any 3)

c) In ΔABC and ΔADC:

1. AC is common	
2. AB = AD	Radii
3. $\hat{C}_2 = \hat{C}_3$	Given
∴ ΔABC ≡ ΔADC	(S, A, S)

d) ΔABC ≡ ΔADC Proved above

$\therefore \hat{A}_3 = \hat{A}_2$

2. a)

$\hat{C}_1 = \hat{G}_1$	tan-chord theorem
$\hat{G}_1 = \hat{C}_3$	AG = AC (radii), isosceles Δ
$\therefore \hat{C}_1 = \hat{C}_3$	Both equal to $\hat{G}_1$

b) In ΔABD and ΔADC

1. AB = AC	Radii
2. BD = DC	tangents from same point equal
3. AD is common	
∴ ΔABD ≡ ΔACD	(S,S,S)

		Statement	Reason
	c)	In ΔABF and ΔAFC	
		1. AB = AC	Radii
		2. AF is common	
		3. $\hat{A}_3 = \hat{A}_2$	ΔABD ≡ Δ ADC
		∴ ΔABD ≡ ΔACD	(S, A, S)
		$\hat{B}_1 = \hat{F}_1$	AB = AF (radii), isosceles Δ
		$\hat{F}_2 = \hat{C}_2$	AF = AC (radii), isosceles Δ
		But $\hat{B}_1 = \hat{C}_2$	Proved above - ∴ ΔABD ≡ ΔACD
		∴ $\hat{B}_1 = \hat{F}_2$	Both equal to $\hat{C}_2$
	d)	$\hat{A}_2 = 2\hat{G}_1$	tan-chord theorem OR ext ∠s of a Δ
		But $\hat{G}_1 = \hat{C}_3$	AG = AC (radii), isosceles Δ and $A_2 = 2G_1$
		∴ $\hat{A}_2 = 2\hat{C}_3$	
	e)	$\hat{C}_{1+2} = 90°$	tan ⊥ radius
3.	a)	$\hat{B}_1 = \hat{D} = x$	tan-chord theorem
		$\hat{A}_1 = 2\hat{D} = 2x$	∠ at centre = 2∠ at circumference
	b)	$\hat{C}_1 = \hat{B}_2$	AC = AB (radii), isosceles Δ
		$\hat{A}_1 = 2x$	proven in question a above.
		$\hat{A}_1 + \hat{C}_1 + \hat{B}_2 = 180°$	Sum of angles in triangle = 180°
		∴ $2x + 2\hat{C}_1 = 180$	
		∴ $2\hat{C}_1 = 180 - 2x$	
		∴ $\hat{C}_1 = 90 - x$	
	c)	$\hat{B}_2 = 90 - x$	$\hat{B}_2 = \hat{C}_1$
		$\hat{B}_1 + \hat{B}_2 = x + 90 - x$	
		∴ $\hat{B}_{1+2} = 90$	
	d)	$\hat{B}_{1+2} = 90$	Proved in question c above.
		∴ line through centre of circle that makes 90° = diameter	
4.	a)	$\hat{G}_1 = \hat{D}_2$	∠s in same seg
		$\hat{D}_2 = \hat{E}_1$	AD = AE (radii), isosceles Δ
		∴ $\hat{G}_1 = \hat{E}_1$	Both equal to $\hat{D}_2$

	b)	AE = AD	radii
		DC = EC	tangents from same point equal
		$\hat{E}_{1+2} = 90$	radius ⊥ tangent
		$\hat{D}_{2+3} = 90$	radius ⊥ tangent
		In ΔAEF and ΔAFD	
		1. AD = AE	radii
		2. AF is common	
		3. $\hat{D}_2 = \hat{E}_1$	AD = AE, isosceles Δ
		∴ ΔAEF ≡ ΔAFD	
		∴ AF ⊥ DE	DF = FE (proven, ΔAEF ≡ ΔAFD), line from centre to midpoint ⊥
		∴ Diagonals bisect at 90°	
		∴ AECD is a square	
	c)	$\hat{G}_1 = \hat{E}_1$	Both equal to $\hat{D}_2$
		∴ GH ∥ ED	Alt ∠s are equal
	d)	$\hat{A}_{3+4} = 90°$	AECD is a square – proved question b above.
		AH = AD	radii
		AG = AE	radii
		∴ Diagonals bisect at 90° and ∴ GHED is a square.	
	e)	In ΔCEF and ΔCDF	
		1. EC = DC	tangents from same point equal
		2. FC is common	
		3. DF = FE	ΔAEF ≡ ΔAFD (proven in question b)
		∴ ΔCEF ≡ ΔCDF	
5.	a)	$\hat{E}_{1+2} = \hat{C}_{1+2}$	∠s in same seg
	b)	AG ⊥ DE	GD = GE (given), ∴ line from centre to midpoint ⊥
		∴ $\hat{G}_4 = 90°$	
		AF ⊥ CB	CF = FB (given), ∴ line from centre to midpoint ⊥
		∴ $\hat{F}_2 = 90°$	
		∴ $\hat{G}_4 = \hat{F}_2$	Both equal to 90°
		∴ CEFG is a cyclic quad	∠s in alt segments =

	c)	$\hat{E}_{1+2} = \hat{G}_4$	Both equal to 90°
		∴ EB ∥ AC	Alt ∠s equal
	d)	$\hat{B}_1 = \hat{C}_1$	EB ∥ AC, alt ∠
		$\therefore \hat{B}_1 = \hat{B}_2$	Both equal to $\hat{C}_1$
	e)	$\hat{B}_{1+2} = \hat{E}_1$	AB = AE (radii), isosceles Δ
		$\hat{E}_{1+2} = 90°$	proved above
		But $\hat{E}_2 = \hat{B}_2$	Both equal to $\hat{C}_1$ (∠s in alt segments – proved above)
		And $\hat{B}_1 = \hat{B}_2$	Proved above
		$\therefore \hat{E}_1 = 90 - \hat{E}_2$	
		$\hat{B}_1 + \hat{B}_2 = 90 - \hat{E}_2$	$\hat{B}_{1+2} = \hat{E}_1$
		$\therefore \hat{B}_1 + \hat{B}_2 + \hat{E}_2 = 90$	
		$\therefore \hat{B}_2 + \hat{B}_2 + \hat{B}_2 = 90 \quad \hat{B}_1 = \hat{B}_2 = \hat{E}_2$	
		$\therefore 3\hat{B}_2 = 90°$	
		$\therefore \hat{B}_2 = 30°$	
	f)	$\hat{B}_{1+2} = \hat{E}_1 = 60$	$\hat{B}_2 = 30°$
		$\therefore \hat{A}_1 = 180 - 60 - 60 = 60$	sum of ∠s in a Δ = 180
		$\hat{D}_2 = \hat{B}_1$	∠s in alt segments
		$\hat{E}_2 = \hat{D}_1$	AE = AD (radii), isosceles Δ
		$\hat{E}_2 = \hat{B}_2 = \hat{B}_1$	Proved above
		$\therefore \hat{D}_{1+2} = 60°$	
		And $\hat{D}_{1+2} = \hat{C}_2$	AC = AD (radii), isosceles Δ
		$\therefore \hat{C}_2 = 60°$	
		$\therefore \hat{A}_3 = 180 - 60 - 60$	sum of ∠s in a Δ = 180
		$\hat{A}_1 + \hat{A}_2 + \hat{A}_3 = 180$	∠s on a straight line
		$\therefore \hat{A}_2 = 180 - 60 - 60$	
		$\therefore \hat{A}_2 = 60°$	
6.	a)	$\hat{D}_1 = \hat{D}_{2+3} = 90°$	radius ⊥ tangent
		$\hat{C}_{4+5} = \hat{C}_{2+3} = \hat{C}_1 = 90°$	radius ⊥ tangent
		$\hat{F}_{1+2+3} = 90°$	∠ at centre = twice ∠ at circumference (BD is a straight line)

b) 2 methods:

1: $\hat{A}_2 = 2D_2$ — ∠ at centre = twice ∠ at circumference

2: $\hat{F}_3 = \hat{D}_2$ — AF = AD (radii), isosceles Δ

$\hat{A}_2 = \hat{F}_3 + \hat{D}_2$ — Ext ∠ of a Δ

$\therefore \hat{A}_2 = \hat{D}_2 + \hat{D}_2 = 2\hat{D}_2$

c) In ΔAFC and ΔACE

1. AF = AE — radii
2. AC is common
3. EC = CF — ⊥ from centre bisects chord

∴ ΔAFC ≡ ΔAEC (S, S, S)

d) In ΔAHC and ΔACG

1. AG = AH — radii
2. $\hat{A}_1 = \hat{A}_2$ — ∴ ΔAFC ≡ ΔACE
3. AC is common

∴ In ΔAHC ≡ ΔAGC (S, A, S)

e) $\hat{D}_3 = \hat{F}_{1+2}$ — tan chord theorem

f) $\hat{C}_3 = C\hat{H}A$ — AC = AH (radii), isosceles Δ

$\hat{B}_1 = \hat{F}_{1+2}$ — AB = AF (radii), isosceles Δ

$\hat{A}_2 + \hat{B}_1 + \hat{F}_{1+2} = 180°$ — sum of ∠s in Δ = 180

$\therefore \hat{A}_2 + \hat{B}_1 + \hat{B}_1 = 180°$

$\therefore 2\hat{B}_1 = 180 - \hat{A}_2$

And $\hat{A}_2 + \hat{C}_3 + \hat{C}_3 = 180°$ — sum of ∠s in Δ = 180

$\therefore 2\hat{C}_3 = 180 - \hat{A}_2$

$\therefore 2\hat{C}_3 = 2\hat{B}_1$ — both $= 180 - \hat{A}_2$

$\therefore \hat{C}_3 = \hat{B}_1$

∴ BF ∥ CH — corresponding ∠s equal