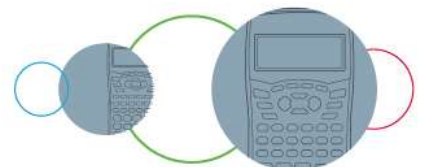


SHARP

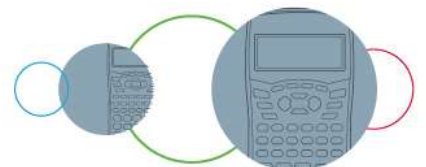
Worksheet 7 Memorandum: Euclidean Geometry

Grade 11 Mathematics

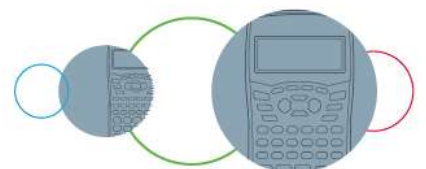
1. a) $\hat{C}_{1+2} = 90^\circ$ Radius \perp Tangent
 $\hat{C}_{3+4} = 90^\circ$ Radius \perp Tangent
 $\therefore \hat{C}_{1+2} = \hat{C}_{3+4}$ Both = 90°
But $\hat{C}_2 = \hat{C}_3$ Given
 $\therefore \hat{C}_1 = \hat{C}_4$
- b) $\hat{B}_2 = \hat{C}_2$ AB = AC (radii), isosceles Δ
 $\hat{B}_1 = \hat{D}_1$ AB = AD (radii), isosceles Δ
 $\hat{C}_3 = \hat{D}_2$ AD = AC (radii), isosceles Δ
 $\hat{C}_2 = \hat{C}_3$ Given
 $\hat{B}_2 = \hat{D}_2$ $\hat{B}_2 = \hat{C}_2 = \hat{C}_3 = \hat{D}_2$ (use any 3)
- c) In ΔABC and ΔADC :
1. AC is common
2. AB = AD Radii
3. $\hat{C}_2 = \hat{C}_3$ Given
 $\therefore \Delta ABC \equiv \Delta ADC$ (S, A, S)
- d) $\Delta ABC \equiv \Delta ADC$ Proved above
 $\therefore \hat{A}_3 = \hat{A}_2$
2. a) $\hat{C}_1 = \hat{G}_1$ tan-chord theorem
 $\hat{G}_1 = \hat{C}_3$ AG = AC (radii), isosceles Δ
 $\therefore \hat{C}_1 = \hat{C}_3$ Both equal to \hat{G}_1
- b) In ΔABD and ΔADC
1. AB = AC Radii
2. BD = DC tangents from same point equal
3. AD is common
 $\therefore \Delta ABD \equiv \Delta ADC$ (S,S,S)



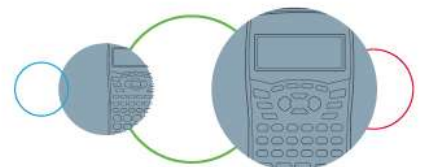
- c) In $\triangle ABF$ and $\triangle AFC$
1. $AB = AC$ Radii
 2. AF is common
 3. $\hat{A}_3 = \hat{A}_2$ $\triangle ABD \equiv \triangle ADC$
- $\therefore \triangle ABD \equiv \triangle ACD$ (S, A, S)
- $\hat{B}_1 = \hat{F}_1$ $AB = AF$ (radii), isosceles \triangle
- $\hat{F}_2 = \hat{C}_2$ $AF = AC$ (radii), isosceles \triangle
- But $\hat{B}_1 = \hat{C}_2$ Proved above - $\therefore \triangle ABD \equiv \triangle ACD$
- $\therefore \hat{B}_1 = \hat{F}_2$ Both equal to \hat{C}_2
- d) $\hat{A}_2 = 2\hat{G}_1$ tan-chord theorem OR ext \angle s of a \triangle
- But $\hat{G}_1 = \hat{C}_3$ $AG = AC$ (radii), isosceles \triangle and $A_2 = 2G_1$
- $\therefore \hat{A}_2 = 2\hat{C}_3$
- e) $\hat{C}_{1+2} = 90^\circ$ tan \perp radius
3. a) $\hat{B}_1 = \hat{D} = x$ tan-chord theorem
- $\hat{A}_1 = 2\hat{D} = 2x$ \angle at centre = $2\angle$ at circumference
- b) $\hat{C}_1 = \hat{B}_2$ $AC = AB$ (radii), isosceles \triangle
- $\hat{A}_1 = 2x$ proven in question a above.
- $\hat{A}_1 + \hat{C}_1 + \hat{B}_2 = 180^\circ$ Sum of angles in triangle = 180°
- $\therefore 2x + 2\hat{C}_1 = 180$
- $\therefore 2\hat{C}_1 = 180 - 2x$
- $\therefore \hat{C}_1 = 90 - x$
- c) $\hat{B}_2 = 90 - x$ $\hat{B}_2 = \hat{C}_1$
- $\hat{B}_1 + \hat{B}_2 = x + 90 - x$
- $\therefore \hat{B}_{1+2} = 90$
- d) $\hat{B}_{1+2} = 90$ Proved in question c above.
- \therefore line through centre of circle that makes $90^\circ =$ diameter
4. a) $\hat{G}_1 = \hat{D}_2$ \angle s in same seg
- $\hat{D}_2 = \hat{E}_1$ $AD = AE$ (radii), isosceles \triangle
- $\therefore \hat{G}_1 = \hat{E}_1$ Both equal to \hat{D}_2



- b) $AE = AD$ radii
 $DC = EC$ tangents from same point equal
 $\hat{E}_{1+2} = 90$ radius \perp tangent
 $\hat{D}_{2+3} = 90$ radius \perp tangent
- In $\triangle AEF$ and $\triangle AFD$
1. $AD = AE$ radii
 2. AF is common
 3. $\hat{D}_2 = \hat{E}_1$ $AD = AE$, isosceles \triangle
- $\therefore \triangle AEF \equiv \triangle AFD$
 $\therefore AF \perp DE$ $DF = FE$ (proven, $\triangle AEF \equiv \triangle AFD$), line from centre to midpoint \perp
- \therefore Diagonals bisect at 90°
 \therefore AECD is a square
- c) $\hat{G}_1 = \hat{E}_1$ Both equal to \hat{D}_2
 $\therefore GH \parallel ED$ Alt \angle s are equal
- d) $\hat{A}_{3+4} = 90^\circ$ AECD is a square – proved question b above.
 $AH = AD$ radii
 $AG = AE$ radii
 \therefore Diagonals bisect at 90° and \therefore GHED is a square.
- e) In $\triangle CEF$ and $\triangle CDF$
1. $EC = DC$ tangents from same point equal
 2. FC is common
 3. $DF = FE$ $\triangle AEF \equiv \triangle AFD$ (proven in question b)
- $\therefore \triangle CEF \equiv \triangle CDF$
5. a) $\hat{E}_{1+2} = \hat{C}_{1+2}$ \angle s in same seg
- b) $AG \perp DE$ $GD = GE$ (given), \therefore line from centre to midpoint \perp
 $\therefore \hat{G}_4 = 90^\circ$
 $AF \perp CB$ $CF = FB$ (given), \therefore line from centre to midpoint \perp
 $\therefore \hat{F}_2 = 90^\circ$
 $\therefore \hat{G}_4 = \hat{F}_2$ Both equal to 90°
 \therefore CEFG is a cyclic quad \angle s in alt segments =



- c) $\hat{E}_{1+2} = \hat{G}_4$ Both equal to 90°
 $\therefore EB \parallel AC$ Alt \angle s equal
- d) $\hat{B}_1 = \hat{C}_1$ $EB \parallel AC$, alt \angle
 $\therefore \hat{B}_1 = \hat{B}_2$ Both equal to \hat{C}_1
- e) $\hat{B}_{1+2} = \hat{E}_1$ $AB = AE$ (radii), isosceles Δ
 $\hat{E}_{1+2} = 90^\circ$ proved above
 But $\hat{E}_2 = \hat{B}_2$ Both equal to \hat{C}_1 (\angle s in alt segments – proved above)
 And $\hat{B}_1 = \hat{B}_2$ Proved above
 $\therefore \hat{E}_1 = 90 - \hat{E}_2$
 $\hat{B}_1 + \hat{B}_2 = 90 - \hat{E}_2$ $\hat{B}_{1+2} = \hat{E}_1$
 $\therefore \hat{B}_1 + \hat{B}_2 + \hat{E}_2 = 90$
 $\therefore \hat{B}_2 + \hat{B}_2 + \hat{B}_2 = 90$ $\hat{B}_1 = \hat{B}_2 = \hat{E}_2$
 $\therefore 3\hat{B}_2 = 90^\circ$
 $\therefore \hat{B}_2 = 30^\circ$
- f) $\hat{B}_{1+2} = \hat{E}_1 = 60$ $\hat{B}_2 = 30^\circ$
 $\therefore \hat{A}_1 = 180 - 60 - 60 = 60$ sum of \angle s in a $\Delta = 180$
 $\hat{D}_2 = \hat{B}_1$ \angle s in alt segments
 $\hat{E}_2 = \hat{D}_1$ $AE = AD$ (radii), isosceles Δ
 $\hat{E}_2 = \hat{B}_2 = \hat{B}_1$ Proved above
 $\therefore \hat{D}_{1+2} = 60^\circ$
 And $\hat{D}_{1+2} = \hat{C}_2$ $AC = AD$ (radii), isosceles Δ
 $\therefore \hat{C}_2 = 60^\circ$
 $\therefore \hat{A}_3 = 180 - 60 - 60$ sum of \angle s in a $\Delta = 180$
 $\hat{A}_1 + \hat{A}_2 + \hat{A}_3 = 180$ \angle s on a straight line
 $\therefore \hat{A}_2 = 180 - 60 - 60$
 $\therefore \hat{A}_2 = 60^\circ$
6. a) $\hat{D}_1 = \hat{D}_{2+3} = 90^\circ$ radius \perp tangent
 $\hat{C}_{4+5} = \hat{C}_{2+3} = \hat{C}_1 = 90^\circ$ radius \perp tangent
 $\hat{F}_{1+2+3} = 90^\circ$ \angle at centre = twice \angle at circumference (BD is a straight line)



- b) 2 methods:
- 1: $\hat{A}_2 = 2D_2$ ∠ at centre = twice ∠ at circumference
 - 2: $\hat{F}_3 = \hat{D}_2$ AF = AD (radii), isosceles Δ
 $\hat{A}_2 = \hat{F}_3 + \hat{D}_2$ Ext ∠ of a Δ
 $\therefore \hat{A}_2 = \hat{D}_2 + \hat{D}_2 = 2\hat{D}_2$
- c) In ΔAFC and ΔACE
1. AF = AE radii
 2. AC is common
 3. EC = CF ⊥ from centre bisects chord
- $\therefore \Delta AFC \equiv \Delta AEC$ (S, S, S)
- d) In ΔAHC and ΔAGC
1. AG = AH radii
 2. $\hat{A}_1 = \hat{A}_2$ $\therefore \Delta AFC \equiv \Delta ACE$
 3. AC is common
- \therefore In ΔAHC \equiv ΔAGC (S, A, S)
- e) $\hat{D}_3 = \hat{F}_{1+2}$ tan chord theorem
- f) $\hat{C}_3 = \hat{C}HA$ AC = AH (radii), isosceles Δ
 $\hat{B}_1 = \hat{F}_{1+2}$ AB = AF (radii), isosceles Δ
 $\hat{A}_2 + \hat{B}_1 + \hat{F}_{1+2} = 180^\circ$ sum of ∠s in Δ = 180
 $\therefore \hat{A}_2 + \hat{B}_1 + \hat{B}_1 = 180^\circ$
 $\therefore 2\hat{B}_1 = 180 - \hat{A}_2$
 And $\hat{A}_2 + \hat{C}_3 + \hat{C}_3 = 180^\circ$ sum of ∠s in Δ = 180
 $\therefore 2\hat{C}_3 = 180 - \hat{A}_2$
 $\therefore 2\hat{C}_3 = 2\hat{B}_1$ both = 180 - \hat{A}_2
 $\therefore \hat{C}_3 = \hat{B}_1$
 $\therefore BF \parallel CH$ corresponding ∠s equal

