SHARP

Worksheet 2 Memo: Equations and Inequalities

Grade 11 Mathematics

1. a) $x^2 + 2x$

$= x^2 + 2x + 1 - 1$

$= (x+1)^2 - 1$

b) $x^2 + 5x$ (R)

$= x^2 + 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2$

$= \left(x + \frac{5}{2}\right)^2 - 6\frac{1}{4}$

c) $3x^2 + 12x$

$= 3(x^2 + 4x)$

$= 3(x^2 + 4x + 4 - 4)$

$= 3(x+2)^2 - 12$

d) $\frac{1}{2}x^2 + x$ (R/C)

$= \frac{1}{2}(x^2 + 2x)$

$= \frac{1}{2}(x^2 + 2x + 1 - 1)$

$= \frac{1}{2}(x+1)^2 - \frac{1}{2}$

e) $x^2 - 3x + 4$

$= x^2 - 3x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 4$

$= \left(x - \frac{3}{2}\right)^2 + 1\frac{3}{4}$

f) $x^2 - 4x + 5$ (R)

$= x^2 - 4x + 4 - 4 + 5$

$= (x-2)^2 + 1$

g) $3x^2 + 7x - 9$

$= 3\left(x^2 + \frac{7}{3}x - 3\right)$

$= 3\left(x^2 + \frac{7}{3}x + \left(\frac{7}{6}\right)^2 - \left(\frac{7}{6}\right)^2 - 3\right)$

$= 3\left[\left(x + \frac{7}{6}\right)^2 - 4\frac{13}{36}\right]$

$= 3\left(x + \frac{7}{6}\right)^2 - 13\frac{1}{12}$

h) $2x^2 - 3x + 5$ (C)

$= 2\left(x^2 - \frac{3}{2}x + \frac{5}{2}\right)$

$= 2\left(x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 + \frac{5}{2}\right)$

$= 2\left[\left(x - \frac{3}{4}\right)^2 + 1\frac{15}{16}\right]$

$= 2\left(x - \frac{3}{4}\right)^2 + 3\frac{7}{8}$

i) $\frac{1}{3}x^2 + 4x - 9$

$= \frac{1}{3}(x^2 + 12x - 27)$

$= \frac{1}{3}(x^2 + 12x + (6)^2 - (6)^2 - 27)$

$= \frac{1}{3}[(x+6)^2 - 63]$

$= \frac{1}{3}(x+6)^2 - 21$

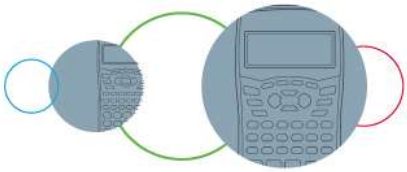
j) $5x^2 - 7x + 6$ (C)

$= 5\left(x^2 - \frac{7}{5}x + \frac{6}{5}\right)$

$= 5\left(x^2 - \frac{7}{5}x + \left(\frac{7}{10}\right)^2 - \left(\frac{7}{10}\right)^2 + \frac{6}{5}\right)$

$= 5\left[\left(x + \frac{7}{10}\right)^2 + \frac{71}{100}\right]$

$= 5\left(x + \frac{7}{10}\right)^2 + 3\frac{11}{20}$



2. a) $3x^2 - 23x + 14 = 0$

$(x-7)(3x-2) = 0$

$x = 7 \ or \ x = \frac{2}{3}$

b) $x^2 - 19x + 21 = 0$ (C)

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{19 \pm \sqrt{(-19)^2-4(1)(21)}}{2(1)}$

$x = 17.82 \quad or \quad x = 1.18$

c) $x^2 + 5x - 66 = 0$

$(x+11)(x-6) = 0$

$x = -11 \quad or \quad x = 6$

d) $2x^2 - 13x - 24 = 0$ (R)

$(x-8)(2x+3) = 0$

$x = 8 \quad or \quad x = -\frac{3}{2}$

e) $2x^2 - 15x + 30 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{15 \pm \sqrt{(-15)^2-4(2)(30)}}{2(2)}$

$\therefore x$ does not exist

f) $2(x^2 - 5x + 2) = x - 1$ (C)

$2x^2 - 10x + 4 = x - 1$

$2x^2 - 11x + 5 = 0$

$(x-5)(2x-1) = 0$

$x = 5 \quad or \quad x = \frac{1}{2}$

g) $2x(x-3) = 5x - 12$

$2x^2 - 6x = 5x - 12$

$2x^2 - 11x + 12 = 0$

$(x-4)(2x-3) = 0$

$x = 4 \quad or \quad x = \frac{3}{2}$

h) $3x^2 - 13x + 12 = 0$ (C)

$(x-3)(3x-4) = 0$

$x = 3 \quad or \quad x = \frac{4}{3}$

i) $4(5x^2 - x + 1) = 17x$

$20x^2 - 4x + 4 - 17x = 0$

$20x^2 - 21x + 4 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{21 \pm \sqrt{(-21)^2-4(20)(4)}}{2(20)}$

$x = \frac{4}{5} \quad or \quad x = \frac{1}{4}$

j) $x^2 - \frac{1}{2}x - 3 = 0$ (C)

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2-4(1)(-3)}}{2(1)}$

$x = 2 \quad or \quad x = -1\frac{1}{2}$

3. a) $8 - x \leq 3x$

$8 \leq 4x$

$2 \leq x \quad or \quad x \geq 2$

b) $4(x-5) > -3(2-x)$ (R)

$4x - 20 > -6 + 3x$

$x > 14$

c) $3x(x-3) \leq 30$

$3x^2 - 9x - 30 \leq 0$

$x^2 - 3x - 10 \leq 0$

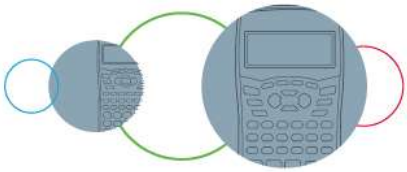
$(x-5)(x+2) \leq 0$

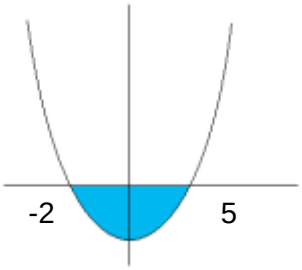
d) $3x(x-1) \geq 2(5x-2)$ (C)

$3x^2 - 3x \geq 10x - 4$

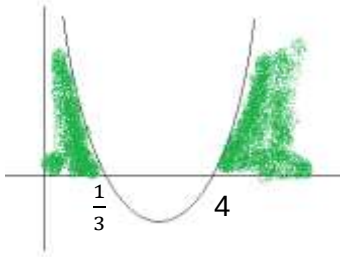
$3x^2 - 13x + 4 \geq 0$

$(3x-1)(x-4) \geq 0$



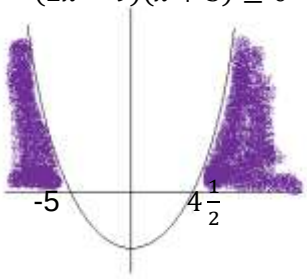


$-2 \leq x \leq 5$



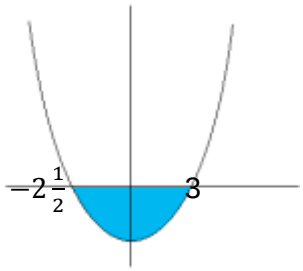
$x \leq \frac{1}{3} \quad or \quad x \geq 4$

e) $2x(x-1) > 3(15-x)$
$2x^2 - 2x \geq 45 - 3x$
$2x^2 + x - 45 \geq 0$
$(2x-9)(x+5) \geq 0$



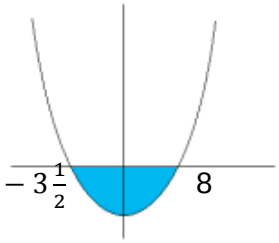
$x \leq -5 \quad or \quad x \geq 4\frac{1}{2}$

f) $2x(x-2) < 3(5-x)$ (C)
$2x^2 - 4x < 15 - 3x$
$2x^2 - x - 15 < 0$
$(2x+5)(x-3) < 0$



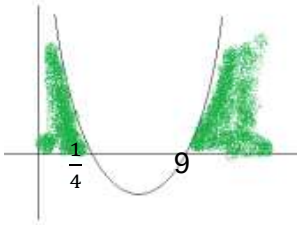
$-2\frac{1}{2} < x < 3$

g) $2x^2 - 3x - 50 \leq 6(x+1)$
$2x^2 - 3x - 50 \leq 6x + 6$
$2x^2 - 9x - 56 \leq 0$
$(2x+7)(x-8) \leq 0$



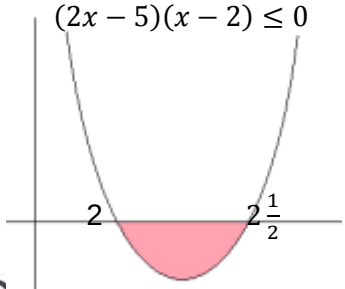
$-3\frac{1}{2} \leq x \leq 8$

h) $4x(x-8) > 5x - 9$ (C)
$4x^2 - 32x - 5x + 9 > 0$
$4x^2 - 37x + 9 > 0$
$(4x-1)(x-9) > 0$

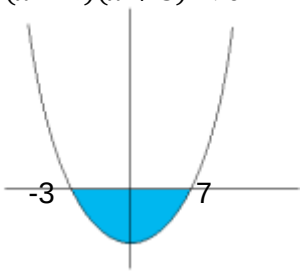


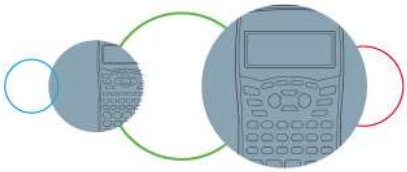
$x < \frac{1}{4} \quad or \quad x > 9$

i) $2x(1-x) \geq 10 - 7x$
$2x - 2x^2 \geq 10 - 7x$
$-2x^2 + 9x - 10 \geq 0$
$2x^2 - 9x + 10 \leq 0$
$(2x-5)(x-2) \leq 0$



j) $x(x+10) < 7(2x-3)$ (C)
$x^2 + 10x < 14x - 21$
$x^2 - 4x + 21 < 0$
$(x-7)(x+3) < 0$





$-3 < x < 7$

$2 \leq x \leq 2\frac{1}{2}$

4. a) $y = -\frac{2}{x+3} + 4 \ldots 1$ and $8y = x + 25$ (C)

$x = 8y - 25 \ldots 2$

Subs 2 into 1

$\therefore y = \frac{-2}{8y-25+3} + 4$

$\therefore y = \frac{-2}{8y-22} + 4$ multiply out by $8y - 22$

$\therefore 8y^2 - 22y = -2 + 32y - 88$

$\therefore 8y^2 - 54y + 90 = 0$

$\therefore (8y - 30)(y - 3) = 0$

$\therefore y = \frac{30}{8} = 3\frac{3}{4}$ *or* $y = 3$ ∴ Subs back into 2:

$x = 8\left(3\frac{3}{4}\right) - 25$ and $x = 8(3) - 25$

$\therefore x = 5$ $x = -1$

$\therefore \left(5; 3\frac{3}{4}\right)$ and (-1; 3)

b) $-y = 2x^2 - 19x + 9$ and $y = -3x + 39 \ldots 1$ (C)

$y = -2x^2 + 19x - 9 \ldots 2$

Subs 1 into 2

$\therefore -3x + 39 = -2x^2 + 19x - 9$

$\therefore 2x^2 - 22x + 48 = 0$

$\therefore x^2 - 11x + 24 = 0$

$\therefore (x - 8)(x - 3) = 0$

$\therefore x = 8$ *or* $x = 3$ ∴ Subs back into 1

$y = -3(8) + 3$ and $y = -3(3) + 39$

$\therefore y = 15$ $y = 30$

∴ (8; 15) and (3; 30)

c) $y = x^2 + x - 30 \ldots 2$ and $y - 2x = 0$ (R)

$y = 2x \ldots 1$

Subs 1 into 2

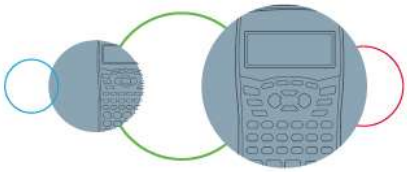
$\therefore 2x = x^2 + x - 30$

$\therefore 0 = x^2 - x - 30$

$\therefore 0 = (x - 6)(x + 5)$

$\therefore x = 6$ *or* $x = -5$ ∴ Subs back into 1

$y = 2(6)$ and $y = 2(-5)$



$\therefore y = 12$ $y = -10$

$\therefore$ (6; 12) and (-5; -10)

d) $y = \frac{3}{x-6} + 1$ …2 and $x - y = 3$ (C)

$x = y + 3$ …1

Subs 1 into 2

$\therefore y = \frac{3}{y+3-6} + 1$

$\therefore y = \frac{3}{y-3} + 1$

$\therefore y^2 - 3y = 3 + y - 3$

$\therefore y^2 - 4y = 0$

$\therefore y(y-4) = 0$

$\therefore y = 0 \quad or \quad y = 4$ $\therefore$ Subs back into 1

$x = 0 + 3$ and $x = 4 + 3$

$\therefore x = 3$ $x = 7$

$\therefore$ (3; 0) and (7; 4)

e) $y = \frac{8}{x-1} - 5$ …2 and $\frac{1}{2}y = 2x - 2\frac{1}{2}$ (C)

$y = 4x - 5$ …1

Subs 1 into 2

$\therefore 4x - 5 = \frac{8}{x-1} - 5$

$\therefore (4x-5)(x-1) = 8 - 5(x-1)$

$\therefore 4x^2 - 4x - 5x + 5 = 8 - 5x + 5$

$\therefore 4x^2 - 4x - 8 = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore (x-2)(x+1) = 0$

$\therefore x = 2 \quad or \quad x = -1$ $\therefore$ Subs back into 1

$y = 4(2) - 5$ and $y = 4(-1) - 5$

$\therefore y = 3$ $y = -9$

(2; 3) and (-1; -9)

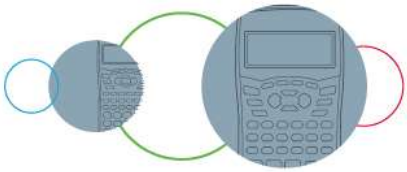
5. Given $y = (x+3)(x^2-2)(x^2+3)$, determine the values of x that are:

a) real $\rightarrow x = -3 \quad or \quad x = \pm\sqrt{2}$ (R)

b) irrational $\rightarrow x = \pm\sqrt{2}$ (R)

c) non-real $\rightarrow x = \sqrt{-3}$ (R)

d) integer $\rightarrow x = -3$ (R)



6. a) $x^2 + x - 1$

$\therefore \Delta = b^2 - 4ac$

$\therefore \Delta = (1)^2 - 4(1)(-1)$

$\therefore \Delta = 1 + 4 = 5$

$\therefore$ roots are real and irrational

b) $2x^2 + 3x - 5$ (R)

$\therefore \Delta = b^2 - 4ac$

$\therefore \Delta = (3)^2 - 4(2)(-5)$

$\therefore \Delta = 9 + 40 = 49$

$\therefore$ roots are real and rational

c) $x^2 - x - 30$

$\therefore \Delta = b^2 - 4ac$

$\therefore \Delta = (-1)^2 - 4(1)(-30)$

$\therefore \Delta = 1 + 120 = 121$

$\therefore$ roots are real and rational

d) $3x^2 + 5$ (R)

$\therefore \Delta = b^2 - 4ac$

$\therefore \Delta = (0)^2 - 4(3)(5)$

$\therefore \Delta = 0 - 60 = -60$

$\therefore$ roots are not real

e) $5x^2 - 43x + 24$

$\therefore \Delta = b^2 - 4ac$

$\therefore \Delta = (-43)^2 - 4(5)(24)$

$\therefore \Delta = 1849 - 480 = 1369 \; or \; 37^2$

$\therefore$ roots are real and rational

f) $x^2 - 2x + 1$ (R)

$\therefore \Delta = b^2 - 4ac$

$\therefore \Delta = (-2)^2 - 4(1)(1)$

$\therefore \Delta = 4 - 4 = 0$

$\therefore$ one real root (equal)

g) $5x^2 - 3x + 43$

$\therefore \Delta = b^2 - 4ac$

$\therefore \Delta = (-3)^2 - 4(5)(43)$

$\therefore \Delta = 9 - 860 = -851$

$\therefore$ roots are not real

h) $x^2 + 4x + 3$ (R)

$\therefore \Delta = b^2 - 4ac$

$\therefore \Delta = (4)^2 - 4(1)(3)$

$\therefore \Delta = 16 - 12 = 4$

$\therefore$ roots are real and rational

i) $x^2 + 3x + 6$

$\therefore \Delta = b^2 - 4ac$

$\therefore \Delta = (3)^2 - 4(1)(6)$

$\therefore \Delta = 9 - 24 = -15$

$\therefore$ roots are not real

j) $x^2 - 4$ (R)

$\therefore \Delta = b^2 - 4ac$

$\therefore \Delta = (0)^2 - 4(1)(-4)$

$\therefore \Delta = 0 + 16 = 16$

$\therefore$ roots are real and rational

7. a) Area of a circle = $\pi x^2 = \pi r^2$

$\therefore x^2 = r^2$

$\therefore x = r$

Area $= l \times b$

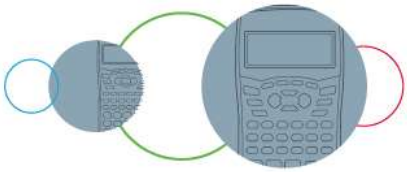
$6 = (x + 5)(x)$

$6 = x^2 + 5x$

$\therefore 0 = x^2 + 5x - 6$

$\therefore 0 = (x + 6)(x - 1)$

$\therefore x = -6 \quad or \; x = 1$



$\therefore$ Perimeter $= 2(l + b)$

$= 2(1 + 5 + 1)$ N/A

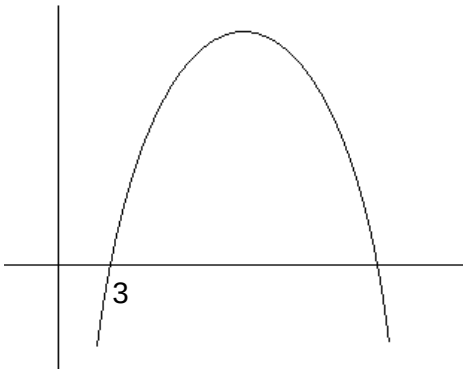
$= 2(7)$

$= 14m$

8. $y = -(x - 3)(x - 6)$

$y = -(x^2 - 9x + 18)$

$y = -x^2 + 9x - 18$ …1



And $2y = 3x - 9$

$\therefore y = \frac{3}{2}x - \frac{9}{2}$ …2

Subs 2 into 1:

$\therefore \frac{3}{2}x - \frac{9}{2} = -x^2 + 9x - 18$

$\therefore x^2 - 7\frac{1}{2}x + 13\frac{1}{2} = 0$

$\therefore 2x^2 - 15x + 27 = 0$ Subs back into 2

$\therefore (2x - 9)(x - 3) = 0$ $\therefore y = \frac{3}{2}\left(\frac{9}{2}\right) - \frac{9}{2}$ and $y = \frac{3}{2}(3) - \frac{9}{2}$

$\therefore x = \frac{9}{2}$ *or* $x = 3$ $\therefore y = 2\frac{1}{4}$ $y = 0$

$\therefore \left(4\frac{1}{2}, 2\frac{1}{4}\right)$ is the point where the arrow will hit the ball.

9. $y = -(x^2 - 9x + 18)$

$\therefore y = -\left(x^2 - 9x + \left(\frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 + 18\right)$ $\therefore y = -\left[\left(x - \frac{9}{2}\right)^2 - 2\frac{1}{4}\right]$

$\therefore y = -\left(x - \frac{9}{2}\right)^2 + 2\frac{1}{4}$ $\therefore$ The maximum height of the ball is 2.25m

10. $xy - 2x = 3$ …1 and $\theta = 45°$ $\therefore m = \tan 45 = 1$

$\therefore y = 1x + c$ Subs in (5; 7)

$\therefore 7 = 1(5) + c$ $\therefore c = 2$ $\therefore y = x + 2$ …2

Subs 2 into 1

$x(x + 2) - 2x = 3$

$x^2 + 2x - 2x = 3$

$\therefore x^2 = 3$ $\therefore$ Subs back into 2: $y = 2 - \sqrt{3}$ OR $y = 2 + \sqrt{3}$

$\therefore x = \pm\sqrt{3}$ $\therefore$ The drill will meet the sand at the coordinates $\left(\sqrt{3}; 2 + \sqrt{3}\right)$